Information Approach to the Analysis of Chaotic and Stochastic Processes in Nonlinear Control Systems

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Abstract— **General provisions of a new direction - chaotic and stochastic processes in decisions of the nonlinear differential equations of the open and closed systems are analyzed. More effective ways of recognition of these processes based on the information approach, and definitions of the reasons causing such processes are offered.**

Keywords— chaotic; stochastic, casual processes; entropy; quantity of information; distribution function

I. Introduction

Now the new direction of the works devoted to revealing and the analysis of chaotic processes (ChP), the determined equations arising in decisions both opened, and the closed systems (control systems (CS)) was generated. The basic tool for revealing ChP is computer simulation of the equations of systems and imitation of their parameters.

In the works devoted to the given direction repeatedly it was noticed, that ChP do not differ from stochastic processes (StP). There were attempts to define the signs peculiar ChP that on these signs to distinguish ChP from the StP. However many of the offered signs as are characteristic for the StP. Except this concept the determined equations and ChP are incompatible and consequently occurrence ChP in the determined systems is difficultly perceived. And occurrence ChP in such systems is shown as the fact, but to the reasons causing such fact do not pay attention.

In the report attempt to carry out comparative analysis ChP and the StP on the basis of the information approach becomes and to define the factors causing occurrence ChP in decisions of the determined equations.

II. Problem Statement

As is known physical systems (PhS) control in real conditions generally are exposed to controllable and uncontrollable revolting influences. In that case it is possible to present the CS equations in a following form:

$$\dot{x} = F(x, u, z, a)\,,\ x(0) = x_0\,, \qquad (1)$$

Where u is a vector of operating influences; z is a vector of controllable indignations, a is a vector of factors of the equation at the set structure of the operator of image F. This coefficient accumulates in itself uncontrollable disturbance - $\xi = (\xi_1, \xi_2,..)$ have place in PhS; x_0-initial values x. At determined operator F and unequivocally certain function $z(t)$ и of sizes a, x_0 the equation (1) is considered determined. Such equation is strongly abstracted representation PhS and a special case stochastic equation.

At construction of the equation (1) accept a number of preconditions. Consider more often, that PhS стационарны, functions $z(t)$, $\xi(t)$ are ergodic-stationary processes.

At the known forecast of function $z(t)$, $t_0 \le t \le T$ or its absence from the equation (1) the following turns out:

$$\dot{x} = F(x, u, a)\,,\ x(0) = x_0\,. \qquad (2)$$

Here operator F has other structure, and coefficient a has other value.

If to synthesize the control in the first equation by certain criterion it is possible to receive the law of control of a kind:

$$u(t) = \rho(x(t), z(t-\tau)), \quad t \ge \tau \le T\,, \qquad (3)$$

Where for stationary $z(t)$ $\tau = 0$, and in the second equation we will receive the law of control of a kind:

$$u(t) = \rho(x(t)) \qquad (4)$$

Taking into account control laws accordingly in (1) and (2) the equation of open system turns out:

$$\dot{x} = F(x, a) \qquad (5)$$

In the works devoted to research ChP in closed systems, i.e. in the CS, basically use the determined nonlinear equation of a kind (2) is third order of certain structure, and in open systems use equation of a kind (5) third order with certain structure or their discrete variants.

The operator of image F in the equation (1) expresses a surface in space x, u, z in the equation (2) expresses a surface in space x, u, and in the equation (5) expresses a surface in space x. Often characteristics PhS cover wide area in space of corresponding variables and have difficult enough form. Attempt to approximate all surface of such characteristics PhS one equations has not crowned success. It is connected by that the equations received thus have very difficult structure and consequently small applied the importance. Proceeding from told, the surface of general characteristic ChP breaks into small areas, number and which numbers we will designate ν, N accordingly, and these areas are approximated by the equations concerning simple structure. Capacity of each ν area is defined, proceeding from desirable structure of the equation and degree of complexity of characteristic PhS in this area.

It is necessary to notice, that for everyone PhS, proceeding from technical regulations, the limited number of operating conditions is defined. The vicinity of each such mode makes corresponding working area. And working areas in overwhelming majority of cases do not cover a general characteristic considered PhS, including the CS. Therefore for the system analysis its equation is made only for the working areas which number is much less than total of areas into which general characteristic ФС breaks. Working areas on a surface of a general characteristic of system can located in the neighborhood or is isolated. The system during each moment of time is only in one of working areas and with change of its parameters (x_0, u, and therefore, a vector of factors) there is quantitative and probably qualitative change in its decision (уркация decisions) in the given area. Thus quantitative change in the decision occurs regularly, and qualitative in steps during the moments when there is a qualitative change in roots of the corresponding characteristic equation. Such approach underlies the qualitative analysis of the equations of systems [1,2]. As a result of the qualitative analysis of the decision of the equation of system by its computer simulation and imitation of parameters such values of the last at which arises PhS are defined.

Considering, that known signs are insolvent for qualitative definition of a randomness or stochasticity of the strange processes arising in decisions of the equations of systems, for this purpose more effective approach based on the important characteristic indicator of-entropy of the theory of the information of K.Shennon is offered. Thus, proceeding from the substantial analysis of chaotic and stochastic processes it is shown, that these processes have the identical (stochastic) nature, but different degrees of uncertainty. Here fairly following.

The theorem 1. Degree of uncertainty of the StP always is less, than in ChP.

It is easily possible to prove it proceeding from that that the joint venture has the established static indicators (probabilities, the moments, distribution functions, correlation functions etc.) which carry certain information on these processes, promote their ordering and carrying out of certain operations over them. And the joint venture theory has old history and the fulfilled powerful mathematical apparatus. Therefore it is not casual, that for the analysis of many known in physicist ChP, for example, in macrosystems at molecular and electronic levels, they are transferred to area of the joint venture and are investigated with use of a mathematical apparatus of the last. It is possible to carry Fermi's, Boze's, Einstein's,_Maxwell's, Boltzmann's, etc. works, which have established the laws of distribution. However, here there is a fair question: if ChP it is adequately possible to describe StP means why it not to name the StP.

Thus ChP it is possible to carry to the StP category, the found which statistics contain big uncertainty so statistical hypotheses about them partially or completely are not carried out.

Use of a mathematical apparatus of the StP (methods of probability theory and the mathematical statistics) in each concrete case, for example, certain demands make to a kind of function of distribution of random variables, ergodic-stationary the StP and etc.

It is known, that the StP displays in itself random variables (RV) and casual events since the StP represents time function of RV, and RV is the set of casual events. Therefore it is possible to present the StP as system RV. As such system also it is possible to present a vector random variable.

The information indicator of uncertainty entropy of system RV, and including the StP is defined on the basis of its generalised function of distribution (FD).

For discrete FD systems from two independent RV X_1, X_2 entropy is defined from following expression:

$$H(X_1, X_2) = \sum_{\nu_1=1}^{N_1} \sum_{\nu_2=1}^{N_2} p(x_{1\nu 1}, x_{2\nu 2}) \log p(x_{1\nu_1}, x_{2\nu_2}), \qquad (6)$$

Where $p(x_{1\nu_1}, x_{2\nu_2})$ is joint probability ν_1-th value X_1, ν_2-th values X_2, found from the general FD RV X_1, X_2; ν_i, N_i are numbers and number of discrete values $X_i, i = \overline{1,2}$.

Expression (6) is easily generalised for multidimensional systems and from it expression for independent RV.

Expression (6) also can be written down in a kind:

$$H(X_1, X_2) = H(X_1) + H(X_2 / X_1) = H(X_2) + H(X_1 / X_2). \quad (7)$$

Whence

$$H(X_1 / X_2) = H(X_1, X_2) - H(X_2) = H(X_1) - H(X_1 \cdot X_2) \quad (8)$$

Or

$$H(X_2 / X_1) = H(X_1, X_2) - H(X_1) = H(X_2) - H(X_1 \cdot X_2), \quad (9)$$

Where

$H(X_1 / X_2)$, $H(X_2 / X_1)$ are conditional entropy which express to entropy X_1, X_2 after information reception about X_2 and X_1 accordingly. For these parities are fair:

$$0 \le H(X_1 / X_2) \le H(X_1), \qquad 0 \le H(X_2 / X_1) \le H(X_2). \qquad (10)$$

Value of entropy RV depends on kind FD and its dispersion. For normal FD continuous RV *X* this dependence looks like:

$$H(X) = \log \frac{\sqrt{2\pi}\sigma}{\Delta x}, \qquad (11)$$

and for uniform FD is

$$H(X) = \frac{1}{b - a}, \qquad (12)$$

where σ is root-mean-square deviation *X*; a, b are limits of definition uniform FD; Δx is size of readout X.

For entropy RV X fairly following.

The theorem 2. For random variables X_i, $i = \overline{1,n}$ accepting values on set of set capacity, having different laws of distribution the parity is fair:

$$H(X_1^H) \le H(X_i) \le H(X_n),\ i \ne \overline{1,n}, \qquad (13)$$

Consequence: Entropy normal and uniform FD are limiting for others FD provided that corresponding RV change in set of is defined capacity. The condition (13) allows in each concrete situation with use of intellectual system to find the value of entropy defining borders of section of casual and chaotic processes, the equations (2) arising in decisions, (5).Если such value of entropy to designate through $f_H(x)$ at $H(x) \le f_H(x)$ X concerns the StP, and otherwise to ChP.

As an information indicator of an estimation of stochasticity or a randomness of the strange processes arising in decisions of the equations it is possible to use and the quantity of the information - $J(x)$.Этот an indicator reflects size of reduction of entropy after reception of the information on $H(x)$ and $J(x)$ there is a certain balance. Value $J(x) : f_J(X)$ dividing stochastic and chaotic processes, arising in decisions of the equations (2), (5) is similarly defined. Thus, if значение $J(X) \le f_J(X)$ X concerns the joint venture, and otherwise to ChP.

Other important question in revealing and analysis ХП, in decisions of the determined nonlinear differential equations is definition of the reasons causing such processes.Как has been noted above, ChP in decisions of the equations reveal by computer simulation of the equations and imitation of their parameters as the fact, but do not give an explanation about occurrence of such processes in the determined systems that causes misunderstanding. Here fairly following:

The theorem 3. For occurrence of chaotic or stochastic process in the decision deterministic the nonlinear differential equation influence on the decision of corresponding character is necessary.

For the proof of the theorem it is enough to prove presence of corresponding influences on the decision of the equations. Actually, such influences exist. For their definition it is necessary containing to analyze principles and technique of construction of the equations and a course of the decision of the last. Proceeding from spent such analysis, we will note the following reasons causing corresponding strange processes.

1. Neglect coherences of separate degrees of freedom x_i in the equations at their structural and parametrical identification. In existing practice identification for each degree of freedom is carried out independently with use of the scalar metrics. For this reason the received equations are badly joined, i.e. in joints of the corresponding equations are formed artificial люфты (tolerance zones), that breaks a decision regularity. And with increase in degrees of freedom the number люфтов increases and even faster strangeness degree grows in character of the decision.

2. The errors of identification stimulating bad compatibility of the equations of degrees of freedom.

3. Errors in measurements, coding, transfer, processing of signals and in the problem decision. Such errors, as a rule, are classified as statistical. It is obvious, that the specified reasons can cause strange processes in decisions of the equations, especially in unsteady modes as in these modes the system is supersensitive to indignations. Considering, that number of influences considerably and they are various, it is possible to present them as the generalized influence with a wide spectrum of frequencies. Clearly, that available spectra of frequencies will be with different intensity and in different directions to influence the decision and, thus, to cause difficult chaotic processes in it.

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