On One Non-Smooth Optimal Control Problem Described by Volterra Type of Two-Dimensional Difference Equation

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***Abstract*— An optimal control problem described by a Volterra type nonlinear difference equation and with non-smooth quality test is considered. Necessary optimality conditions are established.**

***Keywords*— Volterra typedifference equation, non-smooth, terminal functional, admissible process, optimal control.**

I. Introduction

In the papers [1,2] and others, different necessary optimality conditions are obtained for optimal control problems described by two-dimensional difference equations of Volterra type. In the present paper the case of non-smooth quality test is studied for a problem on an optimal control of Volterra type systems. Note that different aspects of problems on optimal control of discrete two-parametric systems were studied in [3-7] and etc.

II. Statement of Problem

It is required to minimize the terminal functional

$$S(u) = \Phi(z(t_1, x_1)), \tag{1}$$

under constraints

$$\begin{gathered} u(t,x) \in U \subset R^r, \quad (t,x) \in T \times X = \\ = \{(t,x) : t = t_0, t_0+1, \ldots, t_1,\ x = x_0, x_0+1, \ldots, x_1\} \end{gathered} \tag{2}$$

$$\begin{gathered} z(t,x) = \sum_{\tau=t_0}^{t} \sum_{s=x_0}^{x} f(t,x,\tau,s,z(\tau,s),u(\tau,s)), \\ (t,x) \in T \times X . \end{gathered} \tag{3}$$

Here $\Phi(z)$ is a given scalar function having derivatives in any direction and satisfying the Lipschits condition, t_0, t_1, x_0, x_1 are the given numbers, moreover the differences $t_1 - t_0$ and $x_1 - x_0$ are natural numbers, U is a given non-empty, bounded set, $f(t,x,\tau,s,z,u)$ - is a given n-dimensional vector-function continuous in totality of variables together with partial derivatives with respect to z, $u(t,x)$ is an r - dimensional vector of control actions.

The control function $u = u(t,x)$ satisfying condition (2) is said to be an admissible control, the appropriate process $u(t,x), z(t,x)$ – an admissible process.

III. Solving the Problem

Assuming that $(u(t,x),\ z(t,x))$ is a fixed admissible process, introduce the denotation

$$\begin{aligned} &\Delta_{v(t,x)} f(t_1,x_1,t,x,z(t,x),u(t,x)) = \\ &= f(t_1,x_1,t,x,z(t,x),v(t,x)) - \\ &- f(t_1,x_1,t,x,z(t,x),u(t,x)), \\ &\Delta_{v(t,x)} f(\alpha,\beta,t,x,z(t,x),u(t,x)) = \\ &= f(\alpha,\beta,t,x,z(t,x),v(t,x)) - \\ &- f(\alpha,\beta,t,x,z(t,x),u(t,x)) \end{aligned}$$

$$\ell(v) = \sum_{t=t_0}^{t_1} \sum_{s=x_0}^{x_1} \Big[\Delta_{v(t,x)} f(t_1,x_1,t,x,z(t,x),u(t,x)) + \\ + \sum_{\alpha=t}^{t_1} \sum_{\beta=x}^{x_1} R(t_1,x_1;\alpha,\beta) \Delta_{v(t,x)} f(\alpha,\beta,t,x,z(t,x),u(t,x))\Big].$$

Here $R(t,x;\tau,s)$ $(n \times n)$ matrix function is the resolvent of the linearized system

$$y(t,x) = \sum_{\tau=t_0}^{t} \sum_{s=x_0}^{x} \big[f_z(t,x;\tau,s,z(\tau,s),u(\tau,s)) + \\ + \Delta_{v(\tau,s)} f(t,x;\tau,s,z(\tau,s),u(\tau,s))\big],$$

that is a solution of $(n \times n)$ matrix difference equation of Volterra type of the form

$$R(m,\ell;\, t,x) = \\ = \sum_{\tau=t}^{m} \sum_{s=x}^{t} R(m,\ell;\, \tau,s) f_z(\tau,s,t,x,z(t,x),u(t,x)) - . \\ - f_z(m,\ell,t,x;z(t,x),u(t,x)).$$

Theorem 1. If along the admissible process $(u(t,x),\ z(t,x))$ the set

$$f(t,x,\tau,s,z(\tau,s),U)=$$
$$\{\alpha:\alpha=f(t,x,\tau,s,z(\tau,s),v),\quad v\in U\}$$

is convex, then for optimality of the admissible control $u(t,x)$ in problem (1)-(3) it is necessary the inequality

$$\frac{\partial\Phi(z(t_1,x_1))}{\partial\ell(v)}\geq 0 \qquad (4)$$

be fulfilled for all $v(t,x)\in U$, $(t,x)\in T\times X$.

Assume that, the following condition holds:

$$q(v)=\sum_{t=t_0}^{t_1}\sum_{x=x_0}^{x_1}\Big[f_u(t_1,x_1,t,x,z(t,x),u(t,x))+$$
$$+\sum_{\alpha=t}^{t_1}\sum_{\beta=x}^{x_1}R(t_1,x_1;\alpha,\beta)f_u(\alpha,\beta,t,x,z(t,x),u(t,x))\Big]\times$$
$$\times(v(t,x)-u(t,x)),$$

Theorem 2. Let the set U be convex, the vector-function $f(t,x,\tau,s,z,u)$ be continuous in totality of variables together with partial derivatives with respect to (z,u). Then for optimality of the admissible control $u(t,x)$ it is necessary the inequality

$$\frac{\partial\Phi(z(t_1,x_1))}{\partial q(v)}\geq 0 \qquad (5)$$

be fulfilled for all $v(t,x)\in U$, $(t,x)\in T\times X$.

Necessary optimality conditions (4) and (5) are rather general. Minimax problem may be investigated by their means. In particular, in the case of smoothness of the function $\Phi(z)$ the discrete maximum principle and linearized maximum principle [7] follows from relations (4) and (5).

IV. Conclusion

In the considered problem, necessary optimality conditions are obtained in the terms of derivatives in directions.

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