Investigation of Singular Controls in Moiseev's Control Problem

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***Abstract*— A problem on optimal control of ordinary dynamic systems with non-standard quality test is considered. A number of necessary optimality conditions are established.**

***Keywords*— Moiseev's control problem; functional; admissible control; necessary optimality conditions; singular controls**

I. Introduction

In the paper [1] N.N. Moiseev considered a problem of optimal control of ordinary dynamic systems with non-standard quality test and proved necessary optimality condition of Pontryagin's maximum principle type. In the papers [2-4] and etc. different necessary optimality condition were obtained for Moiseev type optimal control problems.

In the present paper new optimality conditions are obtained for singular controls in N.N. Moiseev's problem.

II. Statement of Problem

It is required to minimize the functional

$$S_0(u)=\varphi_0(x(t_1))+\int_{t_0}^{t_1}\int_{t_0}^{t_1}F_0(t,s,x(t),x(s))ds\,dt \qquad (1)$$

under constraints

$$u(t)\in U\subset R^r, \qquad t\in T=[t_0,t_1], \qquad (2)$$

$$S_i(u)=\varphi_i(x(t_1))+$$
$$+\int_{t_0}^{t_1}\int_{t_0}^{t_1}F_i(t,s,x(t),x(s))ds\,dt\le 0, \quad i=\overline{1,p}, \qquad (3)$$

$$\dot{x}=f(t,x,u), \quad t\in T, \quad x(t_0)=x_0. \qquad (4)$$

Here $\varphi_i(x)$, $i=\overline{0,p}$ are the given continuously differentiable in R^n scalar functions, $F_i(t,s,a,b)$, $i=\overline{1,p}$ are the given scalar functions continuous in $T\times T\times R^n\times R^n$ together with partial derivatives with respect to (a,b) to second order inclusively, $f(t,x,u)$ is a given n- dimensional vector-function continuous in $T\times R^n\times R^r$ together with partial derivatives with respect to x to second order inclusively, $u(t)$ is an r-dimensional piecewise-continuous (with finite number of first order discontinuity points) vector of control actions with values from the given non-empty and bounded set $U\subset R^r$.

III. Necessary Optimality Condition

Suppose that $(u(t),\ x(t))$ is a fixed admissible process. Introduce the denotation

$$H(t,x,u,\psi_i)=\psi_i'\cdot f(t,x,u)$$
$$\Delta_{v(t)}H^{(i)}(t)\equiv H(t,x(t),v(t),\psi_i(t))-$$
$$-H(t,x(t),u(t),\psi_i(t)),$$
$$\Delta_{v(t)}f(t)\equiv f(t,x(t),v(t))-f(t,x(t),u(t)),$$
$$H_x^{(i)}(t)\equiv H_x(t,x(t),u(t),\psi_i(t)),$$
$$\Delta_{v(t)}H_x^{(i)}(t)\equiv\psi_i'(t)\Delta_v f_x(t)$$
$$\frac{\partial F_i(t,s)}{\partial a}=\frac{\partial F_i(t,s,x(t),x(s))}{\partial a};$$
$$\frac{\partial F_i(t,s)}{\partial b}=\frac{\partial F_i(t,s,x(t),x(s))}{\partial b};$$
$$\frac{\partial^2 F_i(t,s)}{\partial a^2}=\frac{\partial^2 F_i(t,s,x(t),x(s))}{\partial a^2};$$
$$\frac{\partial^2 F_i(t,s)}{\partial a\,\partial b}=\frac{\partial^2 F_i(t,s,x(t),x(s))}{\partial a\,\partial b};$$
$$\frac{\partial^2 F_i(t,s)}{\partial b^2}=\frac{\partial^2 F_i(t,s,x(t),x(s))}{\partial b^2},$$
$$I(u)=\{i: S_i(u)=0,\ i=\overline{1,p}\},\qquad J(u)=\{0\}\cup I(u).$$

Here $v(t)\in U$, $t\in T$ is an arbitrary control function, $\psi_i(t)$ is an n-dimensional vector-function of conjugated variables being a solution of the conjugated problem

$$\dot{\psi}_i=-H_x^{(i)}(t)+\int_{t_0}^{t_1}\left[\frac{\partial F_i(t,s)}{\partial a}+\frac{\partial F_i(s,t)}{\partial b}\right]ds,$$

$$\psi_i(t_1)=-\frac{\partial\varphi_i(x(t_1))}{\partial x},\ i=\overline{0,p}.$$

For simplicity, we'll assume that

$$J(u)=\{0,1,2,\dots,m\}\qquad (m\le p).$$

It holds

Theorem 1. For optimality of the admissible control $u(t)$ it is necessary that the inequality

$$\min_{i\in J(u)}\sum_{j=1}^{m+1}\ell_j\,\Delta_{v_j}H^{(i)}(\theta_j)\le 0 \qquad (5)$$

be fulfilled for all $v_j\in U$, $\ell_j\ge 0$, $\theta_j\in[t_0,t_1]$ $(t_0\le\theta_1\le\theta_2\le\ldots\le\theta_{m+1}<t_1)$.

We can show that inequality (5) is equivalent to the classic maximum condition for problem (1)-(4). But theorem 1 allows to investigate a special case in the considered problem.

Definition 1. The admissible control $u(t)$ is said to be Pontryagin's extremum in problem (1)-(4) if is satisfies optimality condition (5).

Suppose that there exits a non-empty subset $J_0(u)\subset J(u)$ such that for all $i\in J_0(u)$,

$$grad\ \varphi_i(x(t_1))=0,\ \partial F_i(t,s)/\partial a=0,$$
$$\partial F_i(t,s)/\partial b=0.$$

The following fact is true.

Theorem 2. Let $\partial\varphi_i(x(t_1))=0$, $\partial F_i(t,s)/\partial a=0$, $\partial F_i(t,s)/\partial b=0$ for all $i\in J_0(u)$. Then for optimality of Pontyagin's extremum, it is necessary that for any natural number μ the inequality

$$\min_{i\in J(u)}\left\{\sum_{j=1}^{\mu}\sum_{s=1}^{\mu}\ell_j\,\ell_s\,\Delta_{v_j}f(\theta_j)K_i(\theta_j,\theta_s)\Delta_{v_s}f(\theta_s)\right\}\le 0$$

be fulfilled for all $\ell_j\ge 0$, $v_j\in U$, $\theta_j\in[t_0,t_1]$ $(t_0\le\theta_1\le\ldots\le\theta_\mu<t_1)$, $j=\overline{1,\mu}$ such that

$$\min_{i\in J(u)\setminus J_0(u)}\sum_{j=1}^{m}\ell_j\,\Delta_{v_j}H^{(i)}(\theta_j)>0. \qquad (6)$$

Here by definition

$$K(\alpha,\beta)=-\Phi'(t_1,\alpha)\frac{\partial^2\varphi_i(x(t_1))}{\partial x^2}\Phi(t_1,\beta)-$$
$$-\int_{t_0}^{t_1}\left[\int_{\max(\alpha,\beta)}^{t_1}\Phi'(t,\alpha)\frac{\partial^2F_i(t,s)}{\partial a^2}\Phi(t,\beta)dt\right]ds-$$
$$-\int_{\alpha}^{t_1}\left[\int_{\beta}^{t_1}\Phi'(t,\alpha)\frac{\partial^2F_i(t,s)}{\partial a\,\partial b}\Phi(s,\beta)ds\right]dt-$$
$$-\int_{\alpha}^{t_1}\left[\int_{\beta}^{t_1}\Phi'(s,\alpha)\frac{\partial^2F_i(t,s)}{\partial b\,\partial a}\Phi(t,\beta)dt\right]ds-$$
$$-\int_{t_0}^{t_1}\left[\int_{\max(\alpha,\beta)}^{t_1}\Phi'(s,\alpha)\frac{\partial^2F_i(t,s)}{\partial b^2}\Phi(s,\beta)ds\right]dt.$$

IV. CONCLUSION

Necessary optimality conditions are obtained in an optimal control problem with non-standard quality test.

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