Invariant Control Systems of Second Order

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***Abstract*— The external disturbance estimate is used for supporting invariance. Unlimited increase of controller and identifier gain coefficients doesn't lead to stability disturbance of the system. This enables to achieve high speed and accuracy of tracking of standard trajectory. The model problems of stabilization are solved on the packet Matlab/Simulink.**

***Keywords*— invariant control system; disturbance estimation; Lyapunov function; quadratic form; observer**

I. INTRODUCTION

The invariant control systems with respect to external disturbances are more effective in practical applications. The founder of the invariance concept G.V. Shipanov tried to get the absolute invariance condition at the expense of internal connections of the system [1]. Considerably later, in his double-channel principle B.N. Petrov showed that for realization of the invariance principle it is necessary to measure disturbance [2]. The problems of nonlinear nonstationary signals measurement are known well. In some cases it is succeeded to use measurable compounding variables that indirectly characterize external disturbances [3].

In the report, we suggest a method for constructing invariant control systems with respect to external disturbances on the base of disturbance estimation. The use of Utkin-Drazenovich equivalent control [4,5] allowed to get the following distinctive peculiarities:

- Unlimited increase of gain coefficients of the controller and observer doesn't lead to stability disturbance of the system. This enables to achieve high speed and accuracy. Another variants of structures allowing unlimited increase of gain coefficients of the open system were worked out in [6].

- Internal coordinates (state variables) of the system are used for disturbance estimation. Furthermore, by changing the transfer function of the object on the disturbance channel, we have not to tune the compensator. Therefore, unlike the known ones, the systems suggested in the report are called invariant systems of second order.

II. THE PROBLEM END METHOD

Let us consider SISO nonlinear object of n-th order described by the differential equation

$$y^{(n)} = f(t, \boldsymbol{y}) + b(t, \boldsymbol{y})u + \tilde{\varphi}(t), \tag{1}$$

where $\boldsymbol{y} = (y, \dot{y}, \ldots, y^{(n-1)})^T = (x_1, x_2, \ldots, x_n)^T \in R^n$ is the state vector available for measuring; $y \in R$ is controlled output; $u \in R$ is the control; $f(t, \boldsymbol{y})$ is the nonlinear unknown function; $\tilde{\varphi}(t) = \theta(\varphi, \dot{\varphi}, \ddot{\varphi}, \ldots, \varphi^{(m)}),\ m \le n,$ is the uncontrolled external disturbance; $b(t, \boldsymbol{y})$ is the known gain coefficient of the object.

The general problem of stabilization consisting in the selection of the control u that ensures the output motion y(t) of object (1) in the reference path yd(t) after the transient component's termination is considered, where |yd (t)| ≤ d is a sufficiently smooth function. Thus, the system's status traces the reference path

$$\boldsymbol{y}_d = (y_d, \dot{y}_d, \ldots, y_d^{(n-1)})^T.$$

The vector of the tracking error is described by the following expression:

$$\boldsymbol{e} = \boldsymbol{y}_d - \boldsymbol{y} = (e, \dot{e}, \ldots, e^{(n-1)})^T.$$

III. PROBLEM SOLVING

According to the principle of feedback, we will build the control depending on tracking error $\mathbf{e}(t)$.

The control strategy is built on the basis of the fundamental relationship in the Lyapunov function method:

$$dV(\boldsymbol{e})/dt < 0. \tag{2}$$

In the capacity of a challenger for the Lyapunov function, we will accept a quadratic form disintegrating into the linear multipliers:

$$V = \left(a_1 e + a_2 \dot{e} + \ldots + a_n e^{(n-1)}\right)\left(b_1 e + b_2 \dot{e} + \ldots + b_n e^{(n-1)}\right).$$

Accepting $a_i = b_i = c_i,\ i = \overline{1,n},\ and\ c_n = 1,$ after including of the normalizing factor of 1/2, we can write

$$V = 1/2 \cdot s^2 , \quad (3)$$

where

$$s = c_1 e + c_2 \dot{e} + ... + e^{(n-1)}$$

The time derivative of function (3) is as follows:

$$dV/dt = \dot{V} = s\dot{s} , \quad (4)$$

where

$$\dot{s} = c_1 \dot{e} + c_2 \ddot{e} + ... + e^{(n)}.$$

From (4), it follows that, for the realization of relation (2), it is enough to keep up the opposition of the signs s and $\dot{s}$. To this effect, we write $\dot{s} = u_s(s)$.The function u_s responds to the following requirements:

$$\text{if } s > 0 \text{ then } u_s < 0; \text{ if } s < 0 \text{ then } u_s > 0. \quad (5)$$

Depending on the form u_s, it is possible to get various motion modes. For example, when the function is relayed $u_s = -k \cdot sign(s)$, $u_s = -k \cdot sat(s)$, we obtain a sliding motion; when it is a continuous P-, PI- and PID-control, we then obtain asymptotic motion.

From here, we determine the required control u, which can be presented as

$$u = u_{eq} + u_{st} ,$$

where

$$u_{eq} = b(t, \mathbf{y})^{-1} \left(\sum_{i=1}^{n-1} c_i e^{(i)} + y_d^{(n)} - F(t) \right), \quad (6)$$

$$u_{st} = -b(t, \mathbf{y})^{-1} \cdot u_s,$$

$$F(t) = f(t, \mathbf{y}) + \tilde{\varphi}(t).$$

As long as $F(t)$ is difficult to direct the measurement then the realization of u_{eq} the basis of expression (6), we will use estimate $\hat{F}(t)$. Denoting in (6) $F(t) = \hat{F}(t)$. Then, the system's equation in the coordinate s is

$$\dot{s}(t) = -(u_s + \Delta_F), \quad s(0) = s_0, \quad (7)$$

where $\Delta_F = F(t) - \hat{F}(t)$ is the error of observation.

For ensuring the system's stabilization on $s(t)$, it is enough to accept the P-control law:

$$u_s = -k \cdot s(t) , \quad (8)$$

where $k > 0$ is the tuning parameter. Function (8) meets the requirement (5).

The observation problem consists in selection of the estimation $\hat{F}(t)$ under which the influence of $F(t)$ on the solution of system (7) is negligible. As $F(t)$ is unknown and immeasurable, then it is impossible to construct both an astatic and an invariant observer. In this case, one of the simple and effective solution methods of the problem is increase of the gain coefficients of the open-loap. This is possible when the system admits unlimited increase of the gain coefficient without loss of stability.

If for forming $\hat{F}(t)$ we use proportional dependence by the feedback $s(t)$, we can write the observer's equation in the form: $\hat{F}(t) = N \cdot s(t)$, where $N > 0$ is a sufficiently large gain coefficient.

In the steady-state condition $s(t) = -\delta F(t)$, the observer allows to weaken the total influence of nonlinearity and disturbance $\delta = (N + k)$ times.

For realizing the proportional observer, it is enough to form the signal $s = c_1 e + c_2 \dot{e} + ... + e^{(n-1)}$.

There is a specially interesting case when the initial conditions of the standard model and the object coincide. Under the equality $y_d(0) = y(0)$ the considered systems approach simultaneously to "absolutely controlled" and "absolutely invariant " systems. Such systems are characterized by the absence of transient component of the controlled variable $y(t)$ and independence on the external disturbance.

IV. Simulation Results

We consider a problem on stabilization of vibrating object:

$$\ddot{y} = -0.5\dot{y} - y + 2u + \tilde{\varphi}(t) ,$$

where

$$f(t, \mathbf{y}) = -0.5\dot{y} - y, b = 2,$$
$$\tilde{\varphi}(t) = \varphi(t) + 0.4\dot{\varphi}(t), \varphi(t) = \sin(10t), \ y_d(t) = 1(t).$$

The transfer function of the compensator:

$$W_u = \frac{2}{s^2 + 0.5s + 1}, \quad W_f = \frac{0.4s + 1}{s^2 + 0.5s + 1}.$$

The transfer function of the compensator:

$$W_u = \frac{W_f}{W_u} = \frac{0.4s + 1}{2} \approx \frac{0.205s + 0.5}{0.01s + 1}.$$

For an ordinary invariant system we used a PID controller:

$$W_p = \frac{k_p s^2 + k_i s + k_d}{s}.$$

The structural scheme of the invariant control system realization on Matlab/Simulink is given in fig. 1.

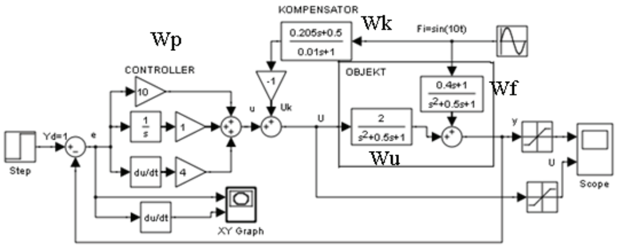
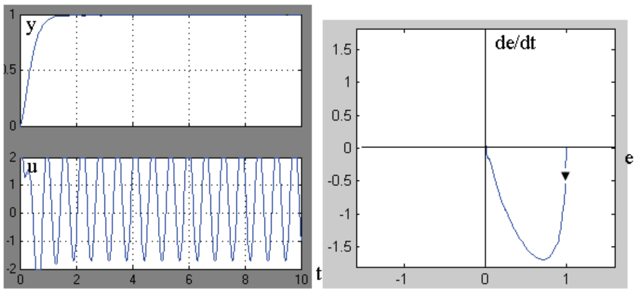


Figure 1. Structural scheme of the invariant system.

Controlled outlet $y(t)$, control signal $u(t)$ (a), and phase portrait of the system (b) are in fig. 2. The tuning parameters $k_p = 10, k_i = 1, k_d = 4$.

As it is seen, the influence of external harmonic disturbance on the outlet was not completely eliminated.



a. b.

Figure. 2. Dynamical characteristics of the system.

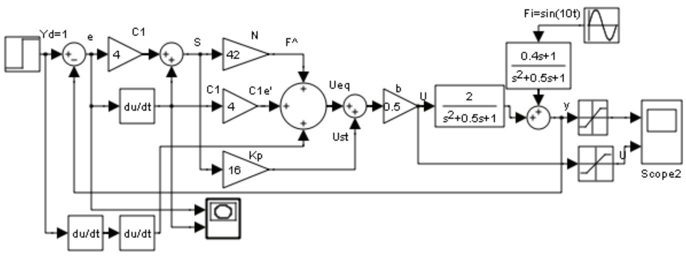
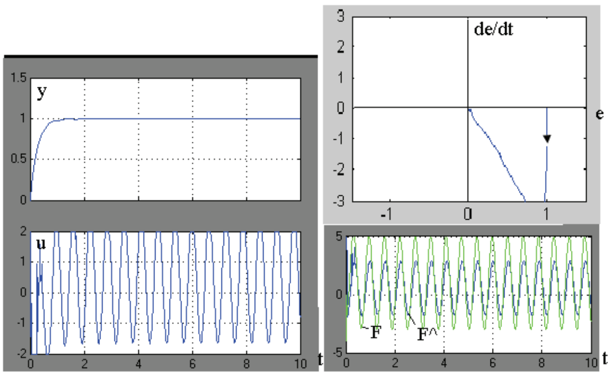


Figure. 3. Structural scheme of the suggested invariant system.



a. b.

Figure. 4. Dynamical characteristics of the system.

The structural scheme of the suggested invariant system is shown in fig. 3. Here

$$u_{eq} = -0.5(c_1 e + \ddot{y}_d(t) + \hat{F}(t)),$$

$$u_{st} = -0.5 k_p \cdot s(t).$$

$$c_1 = 4,\ N = 42,\ k_p = 16.$$

Dynamical characteristics of the system are in fig. 4, a-b.

As it is seen, the quality exponents of transitional characteristics $y_1(t)$ and $y_2(t)$ are almost same. However, in the second case, the external disturbance $\varphi(t)$ is not measured.

V. Conclusions

As a result of references analysis, analytic investigations and computer simulation, we obtained the following main results:

- the suggested condition allows to select the relay, relay-linear and P-,PI-, PID control rules in the capacity of "stabilizing control";
- in the paper, for "stabilizing control" and observer we get proportional P- dependence. Therewith, the structural property of the systems admits unlimited increase of gain coefficients. This ensures high speed and accuracy of tracking of the reference trajectory;
- the stabilizing control and observer are described by rather simple expressions that is very important for their practical realization;
- the regulator has n tuning parameters; one is the gain coefficient of the observer; (n-1) are slopes of the hyperplane. For n=2 we have two parameters that allows to perform tuning manually without special effort. If it is necessary, for determining the tuning parameters, one can use the optimization means [7].

The solution of model problems on the packet Matlab/Simulink enabled to make some positive conclusions of great applied value.

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