Numerical Modeling of Annual Mean Hydrophysical Fields in the Caspian Sea

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Abstract— **The mathematical model of the Caspian Sea, based on a complete system of equations of the sea hydrothermodynamics is considered. The difference scheme of the constructed model is absolutely steady and approximates an initial task with the second order of accuracy on a uniform grid, in space-time coordinates. For the solution of finite-difference equations a splitting method on physical processes and geometrical coordinates is used. The results of numerical experiments with average annual meteorological data and their analysis are indicated.**

Keywords— ***mathematical model; Caspian Sea; hydrothermodynamics; splitting method***

I. INTRODUCTION

Recently, the serious attention of oceanologycians-mathematicians is to the study of the processes, occurring in separate seas, in particular the Caspian Sea [5, 6, 8]. The sea is strongly remote and isolated from the World Ocean, it plays special role in formation of the regional climate and weather. A lot of experimental scientific works is devoted to research of its features.

In the present work numerical model of the Caspian Sea on the basis of the complete system of differential equations of the baroclinic sea hydrothermodynamics is resulted.

II. PROBLEM FORMULATION

Consider the area Ω, possessing cylindrical form, with the lateral Γ surface and bases $z=0$ and $z=H$, where The function H describes the bottom relief of the considered area. The plane xoy of the coordinate system coincides a free surface of the Sea, the axis ox is directed eastward, axis oy – northward, and axis oz – vertically downwards. Then the system of differential equations, describing movement of a liquid, both in approximation of Bussinesque and hydrostatics has a form [1]:

$$\frac{du}{dt} - lv = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + \mu \Delta u + \frac{\partial}{\partial z}\nu\frac{\partial u}{\partial z},$$

$$\frac{dv}{dt} + lu = -\frac{1}{\rho_0}\frac{\partial P}{\partial y} + \mu \Delta v + \frac{\partial}{\partial z}\nu\frac{\partial v}{\partial z},$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \qquad (1)$$

$$\frac{\partial p}{\partial z} = g\rho,$$

$$\frac{dT}{dt} = \mu_T \Delta T + \frac{\partial}{\partial z}\nu_T\frac{\partial T}{\partial z},$$

$$\frac{dS}{dt} = \mu_S \Delta S + \frac{\partial}{\partial z}\nu_S\frac{\partial S}{\partial z},$$

$$\rho = \rho(T, S),$$

where u, v and w are the components of a current speed vector; T, S, P and ρ are the temperature, salinity, pressure and density of marine water; $l=2\omega\sin\varphi$ -Coriolis parameter, ω-angular frequency of the Earth's rotation and φ - geographic latitude; g – gravitational acceleration of the Earth; μ, ν, $\mu_{(T,\ S)}$ and $\nu_{(T,\ S)}$ –coefficients of horizontal and vertical turbulent viscosity and diffusion for momentum, temperature and salinity of a marine water respectively.

Consider the system (1) with the boundary conditions:

$$\nu\frac{\partial u}{\partial z} = -\frac{\tau_x}{\rho_0}, \quad \nu\frac{\partial v}{\partial z} = -\frac{\tau_y}{\rho_0},$$

$$w = 0, \quad T = T_0, \quad S = S_0 \quad \text{at} \quad z = 0,$$

$$u = 0, \quad v = 0, \quad w = 0,$$

$$\frac{\partial T}{\partial z} = 0, \quad \frac{\partial S}{\partial z} = 0 \qquad \text{at} \quad z = H, \qquad (2)$$

$$u = 0,\ v = 0,\ \frac{\partial T}{\partial \vec{n}} = 0, \ \frac{\partial S}{\partial \vec{n}} = 0 \quad \text{on} \quad \Gamma,$$

where τ_x and τ_y are components of atmosphere wind stress; T_0 and S_0- known temperature and salinity distributions on a free surface of the sea.

At the solution of non-stationary problems, besides the boundary conditions (2), we should have initial conditions, too:

$$u = u_0, \ v = v_0, \ T = T_0, \ S = S_0, \quad \text{at} \ t = t_0. \qquad (3)$$

It is necessary to note that the mathematical tasks of existence and uniqueness of solution of the type (1)-(3) problems are in detail considered in works [4] and essential results are achieved. In particular the theorem is proven: If u, v, T and S continuously differentiable of functions of a time, and $u, v, T, S \in C^2(\Omega) \cap C^1(\overline{\Omega})$, $P, w \in C(\Omega)$,

Then the problem (1) – (3) relatively u, v, T, S and w can not have more than one solution, and the function P is determined with accurate to additive constant.

III. METHODS OF SOLUTION

For the solution of the problem (1) – (3) all the time integration interval (0, T) is divided on equal parts and system (1) is linearized on each separate interval $t_n \le t \le t_{n+1}$ according to the work [1-5].

The spatial differential operator of the system (1) is replaced by the finite-difference analogue. For this purpose the area Ω is covered with a difference grid with uniform Δx and Δy steps along axes ox, oy and nonuniform, condensing to the surface Δz_k steps on a vertical, respectively (Fig. 1). In the difference grid Ω^h besides integer knots (i, j, k) we consider intermediates $(i+1/2, j, k)$, $(i, j+1/2, k)$ and $(i, j, k+1/2)$. Thus the continuous function H is replaced by the piece-constant and defined in the integer points (i, j, k) of the grid. Significance of the function H in an intermediate points of the grid are defined as follows:

$$H_{i+1/2,j} = min\ \{ H_{i,j} + H_{i+1,j} \}$$

$$H_{i,j+1/2} = min\ \{ H_{i,j} + H_{i,j+1} \}$$

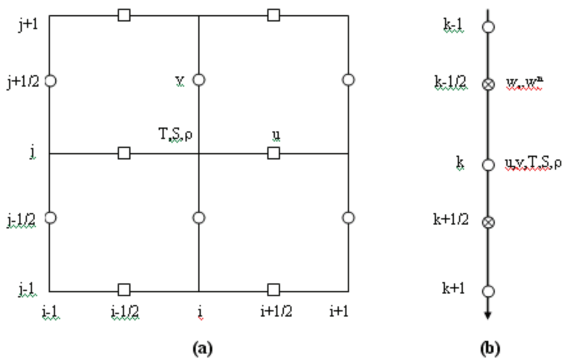

Figure 1. The difference grid of the model.

The finite-difference analogue of the problem is constructed in such manner that it approximates the initial problem with the second order of precision on a uniform grid. Thus in the obtained scheme the same conservation laws are fulfilled to which the differential operator satisfied [4,5].

On the time the Cranc-Nicolson's scheme, with subsequent usage of the methods of split, is used. And the split is done first all on physical processes, then we are come to the problems:

a) problem of a transport-diffusion of momentum, heat and salts,

b) problem of adaptations of the current fields of the fields of masses.

For first of them the method of split on spatial coordinates is applied, then at each partial stage of split are gained linear algebraic systems of the equations with three diagonal matrixes, which are effectively solved by the factorization method.

For the solution of the problem of adaptations of physical fields it is necessary to atmospheric pressure P^a exclude from obtained system. For this purpose in model it is used offered by Marchuk – Bryan technique [1,2], of division of a barotropic component current speed, then problem is parted on two stage. Each of them is solved by an iteration procedure with the subsequent relaxation. The detailed exposition of the relevant algorithms is given in the work [5].

IV. NUMERICAL EXPERIMENTS

On the basis of the above described model the numerical experiments for the Caspian Sea were done. Distance between extreme southern and northern points approximately equals 1200 km, and medial width of the Sea equals 330 km. extreme western point of the Sea is on 46°39′ EL and extreme eastern - 54°44′ EL. The complex bottom configuration of the Caspian Sea is possible to divide into three parts. The isoline 30 m separates on northern part rather extensive shallow-water territory, which average depth does not exceed 10 m. To the south of northern part of the Sea the declination of a bottom configuration begins to be incremented in a direction of Derbent Trough. Here dept of weather reaches 790 m, and to the south the dept again begins to be moderated up to 200 meters on a threshold between Apsheron Peninsula and cape Kuuli. The southern part of the Caspian Sea is characterized by large depts.. Its basic part is occupied by Lenkoran Trough, with the peak dept about 980 meters [3].

The Caspian Sea undergoes rather complex atmospheric action, that is stipulated by its geographical position in moderate latitudes and distance from the World Ocean. Rushing masses, in basic, are three types: continental, Arctic and tropical, and these masses, transiting rater major distance, considerably change the initial physical properties. The features of a local relief introduce additional changes, and consequently, the hydrological features of thermodynamic and physical processes, created by them, are not clear and can describe only schematically.

The numerical experiments were conducted with average annual meteorological data on a free surface. All water area of the sea With the sizes 650 km and

1200 km on longitude and latitude accordingly was covered with a difference grid with uniform $\Delta x \approx 32$ km $\Delta y \approx 31$ km by steps. The number of grid points on a each horizontal plane is equal (20×38). On a vertical the nonuniform grid with the minimal step 2m at a surface and maximal 50 m at a bottom is introduced. 26 levels in total are considered.

The following values of parameters was used in the model:

$\rho_0 = 1$ g. cm^{-3}, $g = 980$ cm.sec^{-2}, $l = l_0 + \beta y$,

$l_0 = 0.95\ _{10}$-4 sec^{-1}, $\beta = 10^{-13}$ cm^{-1}sec^{-1},

$\mu = 5.10^7$cm^{2}sec^{-1}, $\mu_{(T,S)} = 5.10^6$cm^{2}sec^{-1},

Coefficient of a vertical turbulent diffusion is calculated by the Obuchov's formula [8].

$$\nu_{(T,S)} = (0{,}05h)^2 \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 - \frac{g}{\rho_0}\frac{\partial P}{\partial z}},$$

Where the dept of a surface turbulent layer h is determined on the first calculate point z_m, in which condition

$$(0{,}05 z_m)^2 \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 - \frac{g}{\rho_0}\frac{\partial P}{\partial z}} \le \min \nu = 1 \text{ is satisfied.}$$

Below depth h we shall assume, that $\nu_{(T,S)} = 1$. $\nu = 10 * \nu_{(T,S)}$.

As the condition equation the Mamaev's formula is used [7].

$\rho = 1 + 10^{-3}[28{,}152 - 0{,}0735.T - 0,\ 00469.T^2 +$
$+ (0.802 - 0{,}002.T)(S-35)]$.

Data about an atmospheric wind were taken from the atlas of Azerbaijan [3], for summer and winter seasons. With the help of interpolation of these data the fields of an atmospheric wind on the grid used in this model were obtained. The components of friction stress of a wind τ_x, τ_y on known values of wind velocities $\vec{W}(W_x, W_y)$ were calculated by the formulas [4,5]:

$$\tau_x = \nu W_x \left\|\vec{W}\right\|, \quad \tau_y = \nu W_y \left\|\vec{W}\right\|,$$

where

$$\nu = \begin{cases} 0.98448_{10} - 2 & \left\|\vec{W}\right\| \prec 6{,}6 \\ 3.19956_{10} - 2 & \left\|\vec{W}\right\| \ge 6{,}6 \end{cases}$$

Data about a temperature and salinity on a free surface of the Sea were taken from the atlas of Azerbaijan [3].

The initial condition of the sea is set by motionlessness of water weights $u_0(x, y, z)=0$, $v_0(x, y, z)=0$, and averaged values of temperature and salinity $T_0(x, y, z)=\overline{T}(z)$, $S_0(x, y, z)=\overline{S}(z)$ on each horizontal plane.

V. Results and Analisis

The calculation were realized up to establishment of a quasistationary condition, and the expressions $R1 = \frac{1}{|\Omega^h|}\sum_{\Omega^h}(u^2 + v^2)$ and $R2 = \frac{1}{|\Omega^h|}\sum_{\Omega^h}(T^2 + S^2)$ were chosen as the observable functions.

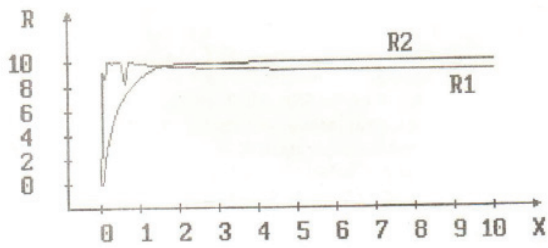

Figure 2. Relation between R1,R2 functions and time t. ($R1_{min}$=0.0, $R1_{max}$=15.6, $R2_{min}$=427.8, $R2_{max}$=458.2.)

On Fig. 2 the curves of functions R1 and R2 are indicated. Nearly quasistationary condition is reached already in two years, and after 10 years the value of functio

$$R = \max\left(\frac{R1^{n+1} - R1^n}{R1^n}, \frac{R2^{n+1} - R2^n}{R2^n}\right)$$

does not exceed 10^{-7} in month. Thus main features of the calculated fields of the currents (Fig. 3), temperature, salinity and density well reflect regularities, known observations, that testifies to ability of model really to reproduce the hydrophisical processes in the Caspian Sea.

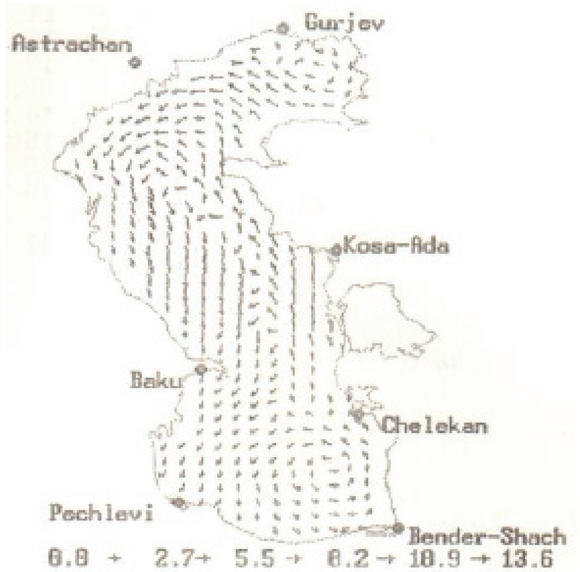

Figure 3. Velocity fields under influence of the average annual data at the surface.

Thus, the submitted model qualitatively well reflect actual physical processes happening in the Caspian Sea and can be successfully used for the solution of various practical problems including ecological tasks.

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