Correlation Matrices in Problems of Identification of Seismic Stability and Technical Condition of High-Rise Buildings and Building Structures

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Abstract— **It is shown that correlation matrices and normalized correlation matrices are applied in solving of static and dynamic identification problems for diagnostics and prediction of technical condition and seismic stability of high-rise buildings and building structures. Specific characteristics and properties of those matrices are determined.**

Keywords— correlation matrices; normalized correlation matrices; static and dynamic identification;, diagnostics; prediction; seismic stability; high-rise buildings; buildings structures

I. INTRODUCTION

It is known that a high-rise building consists of a large number of structural elements, any of which can trigger a train of events resulting in destruction of the whole building. Specialists therefore must be capable of performing precise diagnostics, predicting deformation and vibration of a building or other building structure and forecasting and promptly determining degree of damage. Only maximally quick reliable threat identification and immediate alarming will allow carrying out of evacuation and preventing negative effects caused by gas leakage, fire situations, troubles with electrical equipment, including elevators, etc.

Constant monitoring of all structural elements, meanwhile, is first of all impossible due to technical and economic reasons; on the other hand, it does not produce the desired effect. Thereby, methods of indirect measurements in hard-to-reach spots, modeling and analysis methods, methods of empirical dependences, methods of one-dimensional, two-dimensional and complex multidimensional models are applied to solving of this problem. It allows one to carry out diagnostics of the technical condition of a high-rise building or building structure, predict a failure, determine the moment, at which mechanical deformations exceed threshold values, and detect the beginning of a rescue mission.

On the other hand, most accurate picture of the current condition of a high-rise building or other building structure, such as its wear rate, presence of hidden faults, etc., is obtained after dynamic tests. A full-scale monitoring of technical condition of a high-rise building therefore requires information on its dynamic characteristics and changes. In reality, dynamic probing and early diagnostics of deformation condition of bearing structures are as a rule based on analysis of change in transfer functions built for building sectors of different height. This method is applicable to high-rise building of different architecture, and transfer functions in such cases are built for different sections of the building and constructions.

Transfer function of a building section is understood as correlation of components of power spectra of registered signals in two spots of the building, specifically, at the point of dynamic effect set, for instance, as a broadband impulse from an inelastic impact, and at the point, where this effect is registered after having passed through the part of the building under consideration. Such transfer function characterizes mode of deformation of constructions exactly in the part of the building, which the broadband impulse passed through.

Change in transfer function, in particular, change in values of force coefficients for different frequencies, indicates a change in the mode of deformation in this part of the building, which allows localizing such a change within the number of floors of the building between neighboring measure points.

This, diagnostics and prediction of technical condition and seismic stability of a high-rise building or a building structure require solving of static and dynamic identification problems. The paper considers specifics of the mathematical apparatus applied to solving of those problems.

II. PROBLEM STATEMENT

Solving of static identification problems by probabilistic-statistical methods is known to be reduced to numerical solving of correlation matrix equations.

$$\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(0)\cdot\vec{C}=\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}(0)$$

where

$$\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(0)=\left\|R_{\overset{\circ}{X}_i\overset{\circ}{X}_j}(0)\right\|=\begin{bmatrix} R_{\overset{\circ}{X}_1\overset{\circ}{X}_1}(0) & R_{\overset{\circ}{X}_1\overset{\circ}{X}_2}(0) & \dots & R_{\overset{\circ}{X}_1\overset{\circ}{X}_n}(0) \\ R_{\overset{\circ}{X}_2\overset{\circ}{X}_1}(0) & R_{\overset{\circ}{X}_2\overset{\circ}{X}_2}(0) & \dots & R_{\overset{\circ}{X}_2\overset{\circ}{X}_n}(0) \\ \dots & \dots & \dots & \dots \\ R_{\overset{\circ}{X}_n\overset{\circ}{X}_1}(0) & R_{\overset{\circ}{X}_n\overset{\circ}{X}_2}(0) & \dots & R_{\overset{\circ}{X}_n\overset{\circ}{X}_n}(0) \end{bmatrix},\quad i,j=\overline{1,n}$$

$$\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}(0)=\left\|R_{\overset{\circ}{X}_i\overset{\circ}{Y}}(0)\right\|=\left[R_{\overset{\circ}{X}_1\overset{\circ}{Y}}(0)\quad R_{\overset{\circ}{X}_2\overset{\circ}{Y}}(0)\quad \ldots \quad R_{\overset{\circ}{X}_n\overset{\circ}{Y}}(0)\right]^T$$

$$\vec{C}=[c_1 \quad c_2 \quad \ldots \quad c_n]^T$$

where $\vec{C}$ is column vector of coefficients; $\left\|R_{\overset{0}{X}_i\overset{0}{X}_j}(0)\right\|$ is matrix of estimates of auto- and cross-correlation functions $R_{\overset{\circ}{X}_i\overset{\circ}{X}_i}(0)$, $R_{\overset{\circ}{X}_i\overset{\circ}{X}_j}(0)$ at time shift $\mu=0$ of centered input useful signals $\overset{\circ}{X}_i(i\Delta t)=X_i(i\Delta t)-m_{X_i}$; $\left\|R_{\overset{\circ}{X}_i\overset{\circ}{Y}}(0)\right\|$ is column matrix of estimates of cross-correlation functions $R_{\overset{\circ}{X}_i\overset{\circ}{Y}}(0)$ at time shift $\mu=0$ between centered input useful signal $\overset{\circ}{X}_i(i\Delta t)$ and output signal $\overset{\circ}{Y}(i\Delta t)=Y(i\Delta t)-m_Y$; m_{X_i} and m_Y are mathematical expectations of signals $X_i(i\Delta t)$ and $Y(i\Delta t)$ respectively;

$$R_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)=\frac{1}{N}\sum_{k=1}^{N}\overset{\circ}{X}(k\Delta t)\overset{\circ}{X}((k+\mu)\Delta t)$$

$$R_{\overset{\circ}{X}\overset{\circ}{Y}}(\mu)=\frac{1}{N}\sum_{k=1}^{N}\overset{\circ}{X}(k\Delta t)\overset{\circ}{Y}((k+\mu)\Delta t)$$

However, due to noises $\varepsilon_i(t)$, $\varphi(t)$, which are imposed on useful input signals $X_i(t)$, $i=\overline{1,n}$, correlation matrix equation is transformed to the following form

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)\cdot\vec{B}^*=\vec{R}_{\overset{\circ}{g}\overset{\circ}{\eta}}(0)$$

where

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\left\|R_{\overset{\circ}{g}_i\overset{\circ}{g}_j}(0)\right\|=\begin{bmatrix} R_{\overset{\circ}{g}_1\overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_1\overset{\circ}{g}_2}(0) & \ldots & R_{\overset{\circ}{g}_1\overset{\circ}{g}_n}(0) \\ R_{\overset{\circ}{g}_2\overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_2\overset{\circ}{g}_2}(0) & \ldots & R_{\overset{\circ}{g}_2\overset{\circ}{g}_n}(0) \\ \ldots & \ldots & \ldots & \ldots \\ R_{\overset{\circ}{g}_n\overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_n\overset{\circ}{g}_2}(0) & \ldots & R_{\overset{\circ}{g}_n\overset{\circ}{g}_n}(0) \end{bmatrix},\ i,j=\overline{1,n}$$

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{\eta}}(0)=\left\|R_{\overset{\circ}{g}_i\overset{\circ}{\eta}}(0)\right\|=\left[R_{\overset{\circ}{g}_1\overset{\circ}{\eta}}(0)\quad R_{\overset{\circ}{g}_2\overset{\circ}{\eta}}(0)\quad \ldots \quad R_{\overset{\circ}{g}_n\overset{\circ}{\eta}}(0)\right]^T$$

Meanwhile, general dynamic identification problem is reduced to calculation of impulse transition function $k(\tau)$ or transfer function $W(\tau)$ as a result of certain kind of effect on the input and measuring the system response. Statistical methods of determining of transfer function or impulse transition function are based on the integral equation

$$R_{\overset{\circ}{X}\overset{\circ}{Y}}(\tau)=\int_0^\infty R_{\overset{\circ}{X}\overset{\circ}{X}}(\tau-\lambda)k(\lambda)d\lambda,\ -\infty<\tau<\infty$$

which allows one, using correlation function $R_{\overset{\circ}{X}\overset{\circ}{X}}(\tau)$ of signal $X(t)$ at the input of the construction object and cross-correlation function $R_{\overset{\circ}{X}\overset{\circ}{Y}}(\tau)$ between the output $Y(t)$ and the input, to determine impulse transition function.

Assume that the construction object is in the normal state, and useful signals $X(t)$, $Y(t)$ are received from vibration, motion, tilt, deformation and other sensors. Then solving of statistical dynamic of the construction object can be reduced to solving of a system of equations, which looks as follows in matrix representation:

$$\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}(\mu)=\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)\,\vec{W}(\mu),\ \mu=0,\quad \Delta t,\quad 2\Delta t,\quad \ldots,\quad (N-1)\Delta t$$

where $\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)$ is square symmetric matrix of auto-correlation functions of dimension $N\times N$ of input signal $\overset{\circ}{X}(t)=X(t)$- m_X; $\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}(\mu)$ is column vector of cross-correlation functions between the input $\overset{\circ}{X}(t)$ and output $\overset{\circ}{Y}(t)=Y(t)=m_Y$, m_X, m_Y are mathematical expectations of $X(t)$, $Y(t)$ respectively; $\vec{W}(\mu)$ is column vector of impulse transition function [3].

Correlation matrices $\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)$, $\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}(\mu)$ and column vector of impulse transition functions $\vec{W}(\mu)$ in this case have the following:

$$\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)=\begin{Vmatrix} R_{\overset{\circ}{X}\overset{\circ}{X}}(0) & R_{\overset{\circ}{X}\overset{\circ}{X}}(\Delta t) & \ldots & R_{\overset{\circ}{X}\overset{\circ}{X}}[(N-1)\Delta t] \\ R_{\overset{\circ}{X}\overset{\circ}{X}}(\Delta t) & R_{\overset{\circ}{X}\overset{\circ}{X}}(0) & \ldots & R_{\overset{\circ}{X}\overset{\circ}{X}}[(N-2)\Delta t] \\ \ldots & \ldots & \ldots & \ldots \\ R_{\overset{\circ}{X}\overset{\circ}{X}}[(N-1)\Delta t] & R_{\overset{\circ}{X}\overset{\circ}{X}}[(N-2)\Delta t] & \ldots & R_{\overset{\circ}{X}\overset{\circ}{X}}(0) \end{Vmatrix}$$

$$\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}(\mu)=\left[R_{\overset{\circ}{X}\overset{\circ}{Y}}(0)\quad R_{\overset{\circ}{X}\overset{\circ}{Y}}(\Delta t)\quad \ldots \quad R_{\overset{\circ}{X}\overset{\circ}{Y}}[(N-1)\Delta t]\right]^T$$

$$\vec{W}(\mu)=[W(0) \quad W(\Delta t) \quad \dots \quad W((N-1)\Delta t)]^T$$

When corresponding noises $\varepsilon(t)$, $\varphi(t)$ are imposed on useful signals $X(t)$, $Y(t)$, correlation matrices take the following form:

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\begin{Vmatrix} R_{\overset{\circ}{g}\overset{\circ}{g}}(0) & R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & R_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{Vmatrix}$$

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{\eta}}(\mu)=\left[R_{\overset{\circ}{g}\overset{\circ}{\eta}}(0) \quad R_{\overset{\circ}{g}\overset{\circ}{\eta}}(\Delta t) \quad \dots \quad R_{\overset{\circ}{g}\overset{\circ}{\eta}}[(N-1)\Delta t]\right]^T$$

Thus, dynamic identification problem, as well as static identification problem, is reduced to solving of a symmetric system of linear algebraic equations by means of correlation matrices. Let us therefore consider properties and specifics of correlation matrices in more detail.

III. Technology of Building Correlation Matrices and Their Properties in Problems of Static and Dynamic Identification of Technical Condition and Seismic Stability of High-Rise Buildings and Building Structures

As is known, a correlation matrix is a matrix composed of estimates of auto- and cross-correlation functions.

In static identification problem, system $n \times n$ of correlation functions calculated at time shift $\mu = 0$ and placed in a rectangular table with n rows and n columns

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\begin{bmatrix} R_{\overset{\circ}{g}_1\overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_1\overset{\circ}{g}_2}(0) & R_{\overset{\circ}{g}_1\overset{\circ}{g}_3}(0) & \dots & R_{\overset{\circ}{g}_1\overset{\circ}{g}_n}(0) \\ R_{\overset{\circ}{g}_2\overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_2\overset{\circ}{g}_2}(0) & R_{\overset{\circ}{g}_2\overset{\circ}{g}_3}(0) & \dots & R_{\overset{\circ}{g}_2\overset{\circ}{g}_n}(0) \\ \dots & \dots & \dots & \dots & \dots \\ R_{\overset{\circ}{g}_n\overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_n\overset{\circ}{g}_2}(0) & R_{\overset{\circ}{g}_n\overset{\circ}{g}_3}(0) & \dots & R_{\overset{\circ}{g}_n\overset{\circ}{g}_n}(0) \end{bmatrix}$$

is called a correlation static matrix. rows and columns of the above mentioned matrix are called series of correlation matrix.

Normalized correlation static matrix is a matrix composed of normalized estimates of auto- and cross-correlation functions at time shift $\mu = 0$, i.e. of elements of the above mentioned matrix are divided by corresponding values of variances of parameters:

$$\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\begin{bmatrix} \frac{R_{\overset{\circ}{g}_1\overset{\circ}{g}_1}(0)}{D(g_1)} & \frac{R_{\overset{\circ}{g}_1\overset{\circ}{g}_2}(0)}{\sqrt{D(g_1)\cdot D(g_2)}} & \dots & \frac{R_{\overset{\circ}{g}_1\overset{\circ}{g}_n}(0)}{\sqrt{D(g_1)\cdot D(g_n)}} \\ \frac{R_{\overset{\circ}{g}_2\overset{\circ}{g}_1}(0)}{\sqrt{D(g_2)\cdot D(g_1)}} & \frac{R_{\overset{\circ}{g}_2\overset{\circ}{g}_2}(0)}{D(g_2)} & \dots & \frac{R_{\overset{\circ}{g}_2\overset{\circ}{g}_n}(0)}{\sqrt{D(g_2)\cdot D(g_n)}} \\ \dots & \dots & \dots & \dots \\ \frac{R_{\overset{\circ}{g}_n\overset{\circ}{g}_1}(0)}{\sqrt{D(g_n)\cdot D(g_1)}} & \frac{R_{\overset{\circ}{g}_n\overset{\circ}{g}_2}(0)}{\sqrt{D(g_n)\cdot D(g_1)}} & \dots & \frac{R_{\overset{\circ}{g}_n\overset{\circ}{g}_n}(0)}{D(g_n)} \end{bmatrix}$$

$$=\begin{bmatrix} r_{\overset{\circ}{g}_1\overset{\circ}{g}_1}(0) & r_{\overset{\circ}{g}_1\overset{\circ}{g}_2}(0) & \dots & r_{\overset{\circ}{g}_1\overset{\circ}{g}_n}(0) \\ r_{\overset{\circ}{g}_2\overset{\circ}{g}_1}(0) & r_{\overset{\circ}{g}_2\overset{\circ}{g}_2}(0) & \dots & r_{\overset{\circ}{g}_2\overset{\circ}{g}_n}(0) \\ \dots & \dots & \dots & \dots \\ r_{\overset{\circ}{g}_n\overset{\circ}{g}_1}(0) & r_{\overset{\circ}{g}_n\overset{\circ}{g}_2}(0) & \dots & r_{\overset{\circ}{g}_n\overset{\circ}{g}_n}(0) \end{bmatrix},$$

where $D(g_i)$, $D(g_j)$ - $(i=1,2,\dots,n;\ j=1,2,\dots,n)$ are variances of parameters.

In dynamic identification problem, system $n \times n$ of correlation functions calculated at all time shifts μ and placed in a rectangular table with n rows and n columns is called correlation dynamic matrix:

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu) = \begin{bmatrix} R_{\overset{\circ}{g}\overset{\circ}{g}}(0) & R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & R_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{bmatrix}$$

Normalized correlation dynamic matrix is a matrix composed of normalized estimates of auto- and cross-correlation functions at all time shifts μ, i.e. of elements of the above mentioned matrix are divided by variance of corresponding parameter:

$$\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\begin{Vmatrix} \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(0)}{D(g)} & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t)}{D(g)} & \dots & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t]}{D(g)} \\ \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t)}{D(g)} & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(0)}{D(g)} & \dots & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t]}{D(g)} \\ \dots & \dots & \dots & \dots \\ \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t]}{D(g)} & \frac{\sqrt{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t]}}{D(g)} & \dots & \frac{\sqrt{R_{\overset{\circ}{g}\overset{\circ}{g}}(0)}}{D(g)} \end{Vmatrix}=$$

$$=\begin{Vmatrix} r_{\overset{\circ}{g}\overset{\circ}{g}}(0) & r_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & r_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ r_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & r_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & r_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ r_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & r_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & r_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{Vmatrix} \tag{3.4}$$

where $D(g_i)\ D(g_j)\ (i=1,2,\dots,n \quad j=1,2,\dots,n)$ are variances of parameters.

Correlation functions $\vec{R}_{\overset{\circ}{g}_i \overset{\circ}{g}_j}(0)$, $(i=1,2,\dots,n;\ j=1,2,\dots,n)$ and $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)$ $(\mu=0,1,\ 2,\dots,N)$ composing above mentioned matrices respectively are called elements of correlation matrices.

Elements of normalized correlation matrices above mentioned respectively $r_{\overset{\circ}{g}_i \overset{\circ}{g}_i}(0)$, is auto-correlation function, $r_{\overset{\circ}{g}_i \overset{\circ}{g}_j}(\mu)$ is cross-correlation function. Here, the first index i represents the number of element row, and the second j is the number of its column.

For correlation above mentioned matrices abridged notations are often used: $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\left[R_{\overset{\circ}{g}_i \overset{\circ}{g}_j}(0)\right]$ $(i=1,2,\dots,n;\ j=1,2,\dots,n)$, $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\left[R_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)\right]$, $(\mu=0,1,2,\dots,N)$.

Such a matrix, where $n=n$, is called a square matrix of n order. In particular, correlation matrix of $1\times n$ type is called a row vector, and correlation matrix of $n\times 1$ type is called a column vector.

In solving of static identification problems, when there is no correlation between parameters g_i and g_j, correlation matrix has the following form:

$$\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\begin{bmatrix} R_{\overset{\circ}{g}_1 \overset{\circ}{g}_1}(0) & 0 & 0 & \dots & 0 \\ 0 & R_{\overset{\circ}{g}_2 \overset{\circ}{g}_2}(0) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & R_{\overset{\circ}{g}_n \overset{\circ}{g}_n}(0) \end{bmatrix} \tag{3.5}$$

and is called a diagonal matrix. Diagonal normalized correlation matrix will have the following form in solving of static identification problems:

$$\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\begin{bmatrix} \frac{R_{\overset{\circ}{g}_1 \overset{\circ}{g}_1}(0)}{D(g_1)} & 0 & \dots & 0 \\ 0 & \frac{R_{\overset{\circ}{g}_2 \overset{\circ}{g}_2}(0)}{D(g_2)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{R_{\overset{\circ}{g}_n \overset{\circ}{g}_n}(0)}{D(g_n)} \end{bmatrix}$$

Square correlation matrix $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\left[R_{\overset{\circ}{g}_i \overset{\circ}{g}_j}(0)\right]_{n,n}$ is linked to a determinant. In solving of static identification problems, determinant of correlation matrix $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)$ has the following form:

$$\det \vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\begin{Vmatrix} R_{\overset{\circ}{g}_1 \overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_1 \overset{\circ}{g}_2}(0) & R_{\overset{\circ}{g}_1 \overset{\circ}{g}_3}(0) & \dots & R_{\overset{\circ}{g}_1 \overset{\circ}{g}_n}(0) \\ R_{\overset{\circ}{g}_2 \overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_2 \overset{\circ}{g}_2}(0) & R_{\overset{\circ}{g}_2 \overset{\circ}{g}_3}(0) & \dots & R_{\overset{\circ}{g}_2 \overset{\circ}{g}_n}(0) \\ \dots & \dots & \dots & \dots & \dots \\ R_{\overset{\circ}{g}_n \overset{\circ}{g}_1}(0) & R_{\overset{\circ}{g}_n \overset{\circ}{g}_2}(0) & R_{\overset{\circ}{g}_n \overset{\circ}{g}_3}(0) & \dots & R_{\overset{\circ}{g}_n \overset{\circ}{g}_n}(0) \end{Vmatrix}$$

In solving of static identification problems, determinant of normalized correlation matrix $\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(0)$ has the following form:

$$\det \vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(0)=\begin{vmatrix} \frac{R_{\overset{\circ}{g}_1 \overset{\circ}{g}_1}(0)}{D(g_1)} & \frac{R_{\overset{\circ}{g}_1 \overset{\circ}{g}_2}(0)}{\sqrt{D(g_1)\cdot D(g_2)}} & \dots & \frac{R_{\overset{\circ}{g}_1 \overset{\circ}{g}_n}(0)}{\sqrt{D(g_1)\cdot D(g_n)}} \\ \frac{R_{\overset{\circ}{g}_2 \overset{\circ}{g}_1}(0)}{\sqrt{D(g_2)\cdot D(g_1)}} & \frac{R_{\overset{\circ}{g}_2 \overset{\circ}{g}_2}(0)}{D(g_2)} & \dots & \frac{R_{\overset{\circ}{g}_2 \overset{\circ}{g}_n}(0)}{\sqrt{D(g_2)\cdot D(g_n)}} \\ \dots & \dots & \dots & \dots \\ \frac{R_{\overset{\circ}{g}_n \overset{\circ}{g}_1}(0)}{\sqrt{D(g_n)\cdot D(g_1)}} & \frac{R_{\overset{\circ}{g}_n \overset{\circ}{g}_2}(0)}{\sqrt{D(g_n)\cdot D(g_1)}} & \dots & \frac{R_{\overset{\circ}{g}_n \overset{\circ}{g}_n}(0)}{D(g_n)} \end{vmatrix}$$

In solving of dynamic identification problems, determinant of correlation matrix $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)$ has the following form:

$$\det \vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\begin{Vmatrix} R_{\overset{\circ}{g}\overset{\circ}{g}}(0) & R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & R_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & R_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{Vmatrix}.$$

In solving of dynamic identification problems, determinant of normalized correlation matrix $\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)$ has the following form:

$$\det \vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\begin{vmatrix} \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(0)}{D(g)} & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t)}{D(g)} & \dots & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t]}{D(g)} \\ \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t)}{D(g)} & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}(0)}{D(g)} & \dots & \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t]}{D(g)} \\ \dots & \dots & \dots & \dots \\ \frac{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t]}{D(g)} & \frac{\sqrt{R_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t]}}{D(g)} & \dots & \frac{\sqrt{R_{\overset{\circ}{g}\overset{\circ}{g}}(0)}}{D(g)} \end{vmatrix}$$

Matrix is a rectangular array of numbers, and its determinant is a number determined from known rules, that is:

$$\det \vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\sum_{\chi}(-1)^{\chi}\left(R_{\overset{\circ}{g}_1\overset{\circ}{g}_{i_1}}(0),R_{\overset{\circ}{g}_2\overset{\circ}{g}_{i_2}}(0),\ldots,R_{\overset{\circ}{g}_n\overset{\circ}{g}_{i_n}}(0)\right),$$

where sum is applied to all possible permutations $R_{\overset{\circ}{g}_1\overset{\circ}{g}_{i_1}}(0),R_{\overset{\circ}{g}_2\overset{\circ}{g}_{i_2}}(0),\ldots,R_{\overset{\circ}{g}_n\overset{\circ}{g}_{i_n}}(0)$ of elements $1, 2, \ldots, n$ and therefore contains n summands, with $\chi=0$, if permutation is even and $\chi=1$, if permutation is odd.

By norm of correlation matrix $\vec{R}_{gg}(0)=\lfloor R_{g_ig_j}(0)\rfloor$ is meant a real number $\|\vec{R}_{gg}(0)\|$, complying with the following conditions:

a) $\left\|\vec{R}_{gg}(0)\right\|\geq 0$, *and* $\left\|\vec{R}_{gg}(0)\right\|=0$ *only when* $\vec{R}_{gg}(0)=0;$

b) $\left\|\alpha\left(\vec{R}_{gg}(0)\right)\right\|=|\alpha|\left\|\vec{R}_{gg}(0)\right\|$ *($\alpha-$ a number) and, in particular,* $\left\|-\vec{R}_{gg}(0)\right\|=\left\|\vec{R}_{gg}(0)\right\|;$

c) $\left\|\vec{R}_{gg}(0)+B\right\|\leq\left\|\vec{R}_{gg}(0)\right\|+\|B\|;$

d) $\left\|\left(\vec{R}_{gg}(0)\right)B\right\|\leq\left\|\vec{R}_{gg}(0)\right\|\cdot\|B\|;$

By norm of correlation matrix $\vec{R}_{gg}(\mu)=[\vec{R}_{gg}(\mu)]$ is meant a real number $\|\vec{R}_{gg}(\mu)\|$, complying with the following conditions:

a) $\left\|\vec{R}_{gg}(\mu)\right\|\geq 0$, *and* $\left\|\vec{R}_{gg}(\mu)\right\|=0$ *only when* $\vec{R}_{gg}(\mu)=0;$

b) $\left\|\alpha\left(\vec{R}_{gg}(\mu)\right)\right\|=|\alpha|\left\|\vec{R}_{gg}(\mu)\right\|$ *($\alpha-$ a number) and, in particular,* $\left\|-\vec{R}_{gg}(\mu)\right\|=\left\|\vec{R}_{gg}(\mu)\right\|;$

c) $\left\|\vec{R}_{gg}(\mu)+B\right\|\leq\left\|\vec{R}_{gg}(\mu)\right\|+\|B\|;$

d) $\left\|\vec{R}_{gg}(\mu)B\right\|\leq\left\|\vec{R}_{gg}(\mu)\right\|\cdot\|B\|;$

By norm of normalized correlation matrix $\vec{r}_{gg}(0)=\lfloor\vec{r}_{g_ig_j}(0)\rfloor$ is meant a real number $\|\vec{r}_{gg}(0)\|$, complying with the following conditions:

a) $\left\|\vec{r}_{gg}(0)\right\|\geq 0$, *and* $\left\|\vec{r}_{gg}(0)\right\|=0$ only when $\vec{r}_{gg}(0)=0;$

b) $\left\|\alpha\left(\vec{r}_{gg}(0)\right)\right\|=|\alpha|\left\|\vec{r}_{gg}(0)\right\|$ ($\alpha-$ a number) and, in particular, $\left\|-\vec{r}_{gg}(0)\right\|=\left\|\vec{r}_{gg}(0)\right\|;$

c) $\left\|\vec{r}_{gg}(0)+B\right\|\leq\left\|\vec{r}_{gg}(0)\right\|+\|B\|;$

d) $\left\|\left(\vec{r}_{gg}(0)\right)B\right\|\leq\left\|\vec{r}_{gg}(0)\right\|\cdot\|B\|;$

By norm of normalized correlation matrix $\vec{r}_{gg}(\mu)=[\vec{r}_{gg}(\mu=i\Delta t)]$ is meant a real number $\|\vec{r}_{gg}(\mu)\|$, complying with the following conditions:

a) $\left\|\vec{r}_{gg}(\mu)\right\|\geq 0$, *and* $\left\|\vec{r}_{gg}(\mu)\right\|=0$ only when $\vec{r}_{gg}(\mu)=0;$

b) $\left\|\alpha\left(\vec{r}_{gg}(\mu)\right)\right\|=|\alpha|\left\|\vec{r}_{gg}(\mu)\right\|$ ($\alpha-$ a number) and, in particular, $\left\|-\vec{r}_{gg}(\mu)\right\|=\left\|\vec{r}_{gg}(\mu)\right\|;$

c) $\left\|\vec{r}_{gg}(\mu)+B\right\|\leq\left\|\vec{r}_{gg}(\mu)\right\|+\|B\|;$

d) $\left\|\left(\vec{r}_{gg}(\mu)\right)B\right\|\leq\left\|\vec{r}_{gg}(\mu)\right\|\cdot\|B\|;$

$R_{gg}(0),R_{gg}(\mu),r_{gg}(0),r_{gg}(\mu)$ and B are matrices, for which corresponding operation make sense. For square matrix in particular, we have

$$\left\|\left(\vec{R}_{gg}(0)\right)^{P}\right\| \leq \left\|\left(\vec{R}_{gg}(0)\right)\right\|^{P}$$

$$\left\|\left(\vec{R}_{gg}(\mu)\right)^{P}\right\| \leq \left\|\left(\vec{R}_{gg}(\mu)\right)\right\|^{P}$$

$$\left\|\left(\vec{r}_{gg}(0)\right)^{P}\right\| \leq \left\|\left(\vec{r}_{gg}(0)\right)\right\|^{P}$$

$$\left\|\left(\vec{r}_{gg}(\mu)\right)^{P}\right\| \leq \left\|\left(\vec{r}_{gg}(\mu)\right)\right\|^{P}$$

where p is a natural number.

For correlation matrices $\vec{R}_{gg}(0)=\left[R_{gg}(0)_{ij}\right]$, $\vec{R}_{gg}(\mu)=\left[R_{gg}(\mu)_{ij}\right]$ and normalized correlation matrices $\vec{r}_{gg}(0)=\left[r_{gg}(0)_{ij}\right]$, $\vec{r}_{gg}(\mu)=\left[r_{gg}(\mu)_{ij}\right]$, basically three easily calculated norms are considered:

1. $$\left\|\vec{R}_{gg}(0)\right\|_m = \max_i \sum_j \left|R_{gg}(0)_{ij}\right| \ (m\text{- norm});$$

$$\left\|\vec{R}_{gg}(\mu)\right\|_m = \max_i \sum_j \left|R_{gg}(\mu)_{ij}\right| \ (m\text{- norm});$$

$$\left\|\vec{r}_{gg}(0)\right\|_m = \max_i \sum_j \left|r_{gg}(0)_{ij}\right| \ (m\text{- norm});$$

$$\left\|\vec{r}_{gg}(\mu)\right\|_m = \max_i \sum_j \left|r_{gg}(\mu)_{ij}\right| \ (m\text{- norm});$$

2. $$\left\|\vec{R}_{gg}(0)\right\|_l = \max_j \sum_i \left|R_{gg}(0)_{ij}\right| \ (l\text{- norm});$$

$$\left\|\vec{R}_{gg}(\mu)\right\|_l = \max_j \sum_i \left|R_{gg}(\mu)_{ij}\right| \ (l\text{- norm});$$

$$\left\|\vec{r}_{gg}(0)\right\|_l = \max_j \sum_i \left|r_{gg}(0)_{ij}\right| \ (l\text{- norm});$$

$$\left\|r_{gg}(\mu)\right\|_l = \max_j \sum_i \left|r_{gg}(\mu)_{ij}\right| \ (l\text{- norm});$$

3. $$\left\|\vec{R}_{gg}(0)\right\|_k = \sqrt{\sum_{i,j}\left|R_{gg}(0)_{ij}\right|^2} \ (k \text{ - norm});$$

$$\left\|\vec{R}_{gg}(\mu)\right\|_k = \sqrt{\sum_{i,j}\left|R_{gg}(\mu)_{ij}\right|^2} \ (k \text{ - norm})$$

$$\left\|\vec{r}_{gg}(0)\right\|_k = \sqrt{\sum_{i,j}\left|r_{gg}(0)_{ij}\right|^2} \ (k \text{ - norm})$$

$$\left\|\vec{r}_{gg}(\mu)\right\|_k = \sqrt{\sum_{i,j}\left|r_{gg}(\mu)_{ij}\right|^2} \ (k \text{ - norm})$$

IV. INVERSE CORRELATION MATRIX

Inverse correlation matrix of the given one is a correlation matrix, which, being multiplied by the given correlation matrix both from the right and from the left, gives an identity correlation matrix.

For correlation matrices $\vec{R}_{gg}(0)$, $\vec{R}_{gg}(\mu)$ and normalized correlation matrices $\vec{r}_{gg}(0)$, $\vec{r}_{gg}(\mu)$, let us denote inverse matrices by $R^{-1}_{gg}(0)$, $R^{-1}_{gg}(\mu)$, $r^{-1}_{gg}(0)$, $r^{-1}_{gg}(\mu)$ respectively. Then we will have the following by definition:

$$\left(R_{gg}(0)\right)\left(R_{gg}(0)\right)^{-1} = \left(R_{gg}(0)\right)^{-1} R_{gg}(0) = E$$

$$\left(R_{gg}(\mu)\right)\left(R_{gg}(\mu)\right)^{-1} = \left(R_{gg}(\mu)\right)^{-1} R_{gg}(\mu) = E$$

$$\left(r_{gg}(0)\right)\left(r_{gg}(0)\right)^{-1} = \left(r_{gg}(0)\right)^{-1} r_{gg}(0) = E$$

$$\left(r_{gg}(\mu)\right)\left(r_{gg}(\mu)\right)^{-1} = \left(r_{gg}(\mu)\right)^{-1} r_{gg}(\mu) = E$$

where E is identity matrix.

Determination of inverse correlation matrix for the given one is called correlation matrix inversion.

A correlation matrix is called nonsingular, if its determinant is different from zero.

Otherwise, a correlation matrix is called singular. Any nonsingular correlation matrix has an inverse matrix.

Let us build so-called augmented (or union) matrices for matrices $\vec{R}_{gg}(0), \vec{R}_{gg}(\mu)$ and $\vec{r}_{gg}(0), \vec{r}_{gg}(\mu)$:

$$\vec{RA}_{gg}(0)=\begin{bmatrix} RA_{g_1g_1}(0) & RA_{g_2g_1}(0) & \dots & RA_{g_ng_1}(0) \\ RA_{g_1g_2}(0) & RA_{g_2g_2}(0) & \dots & RA_{g_ng_2}(0) \\ \dots & \dots & \dots & \dots \\ RA_{g_1g_n}(0) & RA_{g_2g_n}(0) & \dots & RA_{g_ng_n}(0) \end{bmatrix}$$

where $RA_{g_ig_j}(0)$ are algebraic complements (minors with signs) of corresponding elements $RA_{g_ig_j}(0)$, $(i,\ j=1,2,\dots,n)$;

$$\vec{RA}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\begin{bmatrix} RA_{\overset{\circ}{g}\overset{\circ}{g}}(0) & RA_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ RA_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & RA_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & RA_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{bmatrix}$$

where $\vec{RA}_{gg}(\mu)$ are algebraic complements (minors with signs) of corresponding elements $R_{gg}(\mu)$;

$$\vec{r}\,a_{gg}(0)=\begin{bmatrix} ra_{g_1g_1}(0) & ra_{g_2g_1}(0) & \dots & ra_{g_ng_1}(0) \\ ra_{g_1g_2}(0) & ra_{g_2g_2}(0) & \dots & ra_{g_ng_2}(0) \\ \dots & \dots & \dots & \dots \\ ra_{g_1g_n}(0) & ra_{g_2g_n}(0) & \dots & ra_{g_ng_n}(0) \end{bmatrix}$$

where $ra_{g_ng_n}(0)$ are algebraic complements (minors with signs) of corresponding elements $r_{g_ng_n}(0)$;

$$\vec{r}a_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)=\begin{Vmatrix} ra_{\overset{\circ}{g}\overset{\circ}{g}}(0) & ra_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ ra_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & ra_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & ra_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{Vmatrix}$$

where $ra_{g_ng_n}(\mu)$ are algebraic complements (minors with signs) of corresponding elements $r_{g_ng_n}(\mu)$.

It should be noted that algebraic complements of row elements are placed into corresponding columns, i.e. transposition is performed.

Let us divide all elements of correlation matrix $\vec{R}\,A_{gg}(0)$ by the value of determinant, i.e. by $\det\vec{R}_{gg}(0)$:

$$\vec{R}_{gg}^{\ -1}(0)=\frac{1}{\det\vec{R}_{gg}(0)}\begin{Vmatrix} RA_{g_1g_1}(0) & RA_{g_2g_1}(0) & \dots & RA_{g_ng_1}(0) \\ RA_{g_1g_2}(0) & RA_{g_2g_2}(0) & \dots & RA_{g_ng_2}(0) \\ \dots & \dots & \dots & \dots \\ RA_{g_1g_n}(0) & RA_{g_2g_n}(0) & \dots & RA_{g_ng_n}(0) \end{Vmatrix}$$

Let us divide all elements of correlation matrix $\vec{R}\,A_{gg}(\mu)$ by the value of determinant, i.e. by $\det\vec{R}_{gg}(\mu)$:

$$\vec{R}_{gg}^{\ -1}(\mu)=\frac{1}{\det\vec{R}_{gg}(\mu)}\begin{bmatrix} RA_{\overset{\circ}{g}\overset{\circ}{g}}(0) & RA_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ RA_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & RA_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & RA_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & RA_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{bmatrix}$$

Let us divide all elements of normalized correlation matrix $\vec{r}\,a_{gg}(0)$ by the value of determinant, i.e. by $\det\vec{r}_{gg}(0)$:

$$\vec{r}_{gg}^{\ -1}(0)=\frac{1}{\det\vec{r}_{gg}(0)}\begin{bmatrix} ra_{g_1g_1}(0) & ra_{g_2g_1}(0) & \dots & ra_{g_ng_1}(0) \\ ra_{g_1g_2}(0) & ra_{g_2g_2}(0) & \dots & ra_{g_ng_2}(0) \\ \dots & \dots & \dots & \dots \\ ra_{g_1g_n}(0) & ra_{g_2g_n}(0) & \dots & ra_{g_ng_n}(0) \end{bmatrix}$$

Let us divide all elements of normalized correlation matrix $\vec{r}\,a_{gg}(\mu)$ by the value of determinant, i.e. by $\det\vec{r}_{gg}(\mu)$:

$$\vec{r}_{gg}^{\ -1}(\mu)=\frac{1}{\det\vec{r}_{gg}(\mu)}\begin{bmatrix} ra_{\overset{\circ}{g}\overset{\circ}{g}}(0) & ra_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & \dots & ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] \\ ra_{\overset{\circ}{g}\overset{\circ}{g}}(\Delta t) & ra_{\overset{\circ}{g}\overset{\circ}{g}}(0) & \dots & ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-1)\Delta t] & ra_{\overset{\circ}{g}\overset{\circ}{g}}[(N-2)\Delta t] & \dots & ra_{\overset{\circ}{g}\overset{\circ}{g}}(0) \end{bmatrix}$$

Those are inverse correlation matrices $\vec{R}_{gg}^{\ -1}(0)$, $\vec{R}_{gg}^{\ -1}(\mu)$, $\vec{r}_{gg}^{\ -1}(0)$, $\vec{r}_{gg}^{\ -1}(\mu)$

Thus, reliable prediction of technical condition of a high-rise building or building structure requires application of the correlation matrices described above, which makes it possible to solve problems of control, monitoring, identification, predication, diagnostics and detection of malfunction at early stages.

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