Research of a Class of Smooth Membership Functions and Its Application in Recognition Systems

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***Abstract*— A class of membership functions for fuzzy sets, which generalize known types of membership functions was offered and investigated. Was analyzed effectiveness of the hand-printed recognition system using neuro-fuzzy network with membership functions from different classes. Analyzed results of numerical experiments of recognition system with membership functions from different classes. Revealed comparative advantage of smooth membership functions offered by authors.**

***Keywords*— smooth membership functions; hand-printed recognition system; feature extraction; neuro-fuzzy network**

I. Introduction

Fuzzy models became one of traditional direction in the recognition theory. In particular, for solving some recognition tasks fuzzy models and fuzzy conclusion are used. Decision-making in pattern recognition problems is carried out by using fuzzy conclusion, rules base of fuzzy production, which is automatically constructed as result of analysis of training samples. Different researchers used similar models in systems for recognition of Chinese, Persian, Arabic, Bulgarian texts and also for recognition of hand-printed digits [1], [2], [3]. In this work were analyzed results of the feed forward neural network with linguistic input variables and fuzzy rules for recognition of Azerbaijani hand-printed characters, written by different writers, in different styles (Fig. 1).

Figure 1. Some characters from training database.

In the paper was investigated parametrical class of membership functions $\mu_A(x;P)$, $P=(p_1,p_2,\ldots,p_{12})\in R^{12}$, constructed by a principle of splines-functions [4]. Offered functions, unlike other known membership functions (triangular, trapezoidal, gaussian, bell [5]) are characterized by smoothness and asymmetry and possess following properties:

1) Membership functions everywhere are continuous with their derivatives on both arguments: $(\mu_A(x,P))'_x$, $(\mu_A(x,P))'_P$, and for expression of all derivatives there are simply calculated analytical formulas;
2) For any possible values of a vector of parameters P and argument values $x\in X^0$, $X^0=supp\ \ \mu_A(x,p)=\{x\in X: 0<\mu_A(x;P)\}$ - support ([6]), MF are strictly unimodal functions;
3) Area X^0 generally can be broken into 5 parts: on the ends of area X^0 MF it is convex, in an average part it is bent, the convex and bent parts of membership functions separate with linear functions. In special cases any parts of MF can be absent;
4) For any numerical values of $x\in X^0$ values of MF are various for various parameters P_1, P_2 from any arbitary small interval, i.e. from $\left\|P^2-P^1\right\|_{R^{12}}<\varepsilon$, $P^1\neq P^2$ follows $\mu_A(\bar{x},P^1)\neq\mu_A(\bar{x},P^2)$;

First three properties of an offered class of functions have great importance in "training" problems, in which adjustment of parameters of MF is carried out. Now most widely it is used in training of computer systems for image recognition of different directions constructed on base of neural-fuzzy networks. The training level consists in using of iterative procedures in space of parameters of neural-fuzzy networks based on first-order optimization methods (gradient types), more exactly on back propagation method ([6]). Clearly that in case of using not differentiated MF it is impossible to prove application of these methods. For non-smooth MF it is necessary to use methods of not differentiated optimization which concede to "smooth" methods.

The paper is organized as follows. Section 1 describes construction of membership functions from offered smooth class. Section 2 is about skeletization and different classes of features for character classification; section 3 describes construction of membership functions for feature. In this work different classes of membership functions were used: triangular, trapezoidal, Gaussian, Bell and smooth class of membership functions . In section 4 the structure of neuro-fuzzy set for recognition and training algorithm for this set was described. In section 5 results of numerical experiments of hand-printed recognition system for Azerbaijani texts was shown.

II. Construction of Membership Functions from Smooth Class

For construction offered parametrycal class of functions $\mu_A(x;P)$, where $P \in R^{12}$ - are the parameters of the function, let's break $[\underline{a}, \overline{a}]$ piece (support) with points p_i, $i = 1,...,7$ posessing following conditions:

$$\underline{a} = p_1 \le p_2 \le p_3 \le p_4 = \tilde{a} \le p_5 \le p_6 \le p_7 = \overline{a} \quad (1)$$

Values of MF in these points should satisfy to following conditions:

$$\begin{aligned} &\mu_A(p_1;P) = \mu_A(p_7;P) = 0, \\ &\mu_A(p_i;P) = p_{i+7}, \; i = 2,\dots,6. \end{aligned} \quad (2)$$

For properties 2, 3 should be executing following inequalities:

$$\begin{aligned} &p_{10} > p_9 > p_8, \quad p_{10} > p_{11} > p_{12}, \\ &p^-_{2,1} \cdot p^-_{9,8} \ge p_8 \cdot p^-_{3,2}, \quad p^-_{7,6} \cdot p^-_{11,12} \ge p_{12} \cdot p^-_{6,5}, \\ &p^-_{5,4} \cdot p^-_{11,12} \ge p^-_{10,11} \cdot p^-_{6,5}, \; p^-_{7,6} \cdot p^-_{11,12} \ge p_{12} \cdot p^-_{6,5}, \end{aligned} \quad (3)$$

here the following designation is used: $p^{\pm}_{i,j} = p_i \pm p_j$, $i, j = 1,\dots,12$. Clearly that considering (1), (2), MF will be normalized, when $\mu_A(p_4;P) = p_{10} = 1$, else MF will be not normalized.

Let on each of intervals (p_i, p_{i+1}), $i = 1,...,6$ MF has following view:

$$\mu_A(x;P) = f_i(x, K^i) = k_1^i + k_2^i x + k_3^i / (k_4^i + x), \quad (4)$$

$x \in (p_i, p_{i+1})$, $i = 1,\dots,6$, where $K^i = (k_1^i, k_2^i, k_3^i, k_4^i) \in R^4$ - is coefficient vector, and $k_3^2 = k_3^5 = 0$, because on the second and fifth intervals: (p_2, p_3) and (p_5, p_6) MF is linear. For offered class of membership functions interpolation and approximation tasks were solved [8], some characteristics (α - cut, the weighed average and average representative, width) of fuzzy numbers with membership functions from smooth class were calculated and arithmetical operations on fuzzy numbers with α - cuts and without α - cuts were defined [9].

Zadeh's extension principle is one of the most important tools in fuzzy set theory. It enables to extend any map $h:S_1xS_2x\dots xS_n \to S$, where $S_1, \dots, S_n$ and S are sets, to a map $h^*:F(S_1)x\dots x\ F(S_n) \to F(S)$ where, for every set X, F(X) denotes the class of fuzzy subsets of X; namely the formula to define h^* is

$$h^*(S_1,\dots,S_n)\ (x) = \sup\{\ s_1(x_1) \cap \dots \cap s_n(x_n) / h(x_1,\dots,x_n) = x\}$$

for $s_1 \in F(S_1), \dots, s_n \in F(S_n)$ and $x \in S$.

Let A is the fuzzy number with membership functions with parameter vector P^A and B is the fuzzy number with membership functions with parameter vector P^B. For these fuzzy numbers we can define following arithmetical operations:

a) The parameters of membership function of addition of A and B fuzzy numbers C=A+B calculates as follows:

$$p_i^C = p_i^A + p_i^B,\ i = 1,..,7$$
$$p_i^C = \min(p_i^A, p_i^B),\ i = 8,..,12.$$

b) The parameters of membership function of subtraction of A and B fuzzy numbers C=A-B calculates as follows:

$$p_i^C = p_i^A - p_{7-i}^B,\ i = 1,..,7$$
$$p_i^C = \min(p_i^A, p_{20-i}^B),\ i = 8,..,12.$$

c) The parameters of membership function of multiplication of A and B fuzzy numbers C=AB calculates as follows:

$$p_i^C = p_i^A p_i^B,\ i = 1,..,7$$
$$p_i^C = \min(p_i^A, p_i^B),\ i = 8,..,12.$$

d) The parameters of membership function of division of A and B fuzzy numbers C=A/B calculates as follows:

$$p_i^C = p_i^A / p_{7-i}^B,\ i = 1,..,7$$
$$p_i^C = \min(p_i^A, p_{20-i}^B),\ i = 8,..,12.$$

III. Image Processing And Feature Extraction

A. Skeletization

In this work a line-thinning algorithm (LTA) was used [10].

This method for extracting the skeleton of a picture consists of removing all the contour points of the picture except those points that belong to the skeleton. In fig. 2 was described skeletization of a word combination in Azerbaijani language.

B. The first class of features

Following features, used in this work for character classification were given from literature [11].

- Moment invariants;
- Shape number;
- Hough transform to identifying a line;

C. The second class of features

The second class of features were offered by us [9]:

- The coefficient of occupancy –
$$\theta = (N_i / N) \cdot 100,$$

here N_i – is the number of dark pixels in i^{th} rectangle, N – is the maximum number of pixels in this rectangle;

- Length on an axis x –
$$l_x = \max_{i=1,N_i} |x_i - x_1|;$$
- Length on an axis y –
$$l_y = \max_{i=1,N_i} |y_i - x_1|;$$
- Angle coefficient –
$$k = (y_{N_i} - y_1)/(x_{N_i} - x_1).$$

IV. Membership Functions for Features

For features in each rectangle, which are linguistic variables were defined terms and constructed membership functions from different classes. For example, in first rectangle for the coefficient of occupancy were defined following terms: "more empty", "empty", "full" and "more full". In fig. 3 was shown graphic of smooth membership function for term "full" of feature coefficient of occupancy.

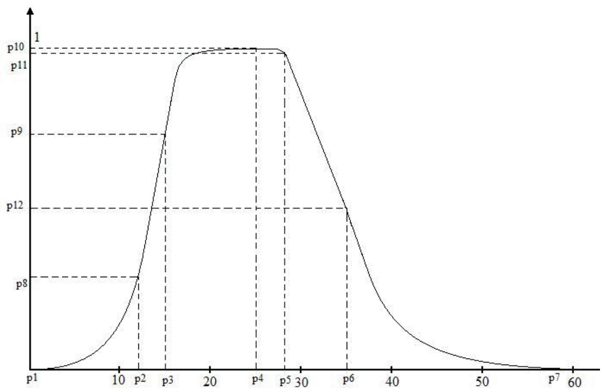

Figure 3. Smooth membership functions for term "full" of feature coefficient of occupancy.

V. Structure and Training of Neuro-Fuzzy Network

For recognition of Azerbaijani characters was constructed hybrid neural network with three layers [12]. Inputs of the network are features, defined after processing and skeletization of given character, in the first layer membership degrees to terms for these features are calculated. Following layer consists of IF-THEN rules for each character. In the last layer is carried out gravity center defuzzification method and defined the number of class, where given character belongs.

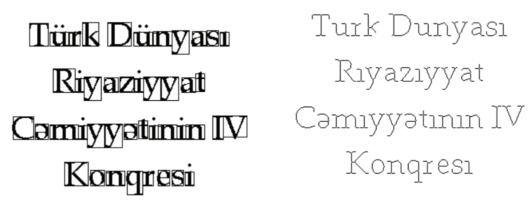

Figure 2. Skeletization of a word combination in Azerbaijani.

For training of neural network error back propagation method was used. For minimizing of error function, which define difference between desired result and result of the neural network for learning character, conjunctive gradient method was applied. In training process were modified parameters of membership functions. The number of optimized parameters for each term differs during training of neural network with membership functions from different classes.

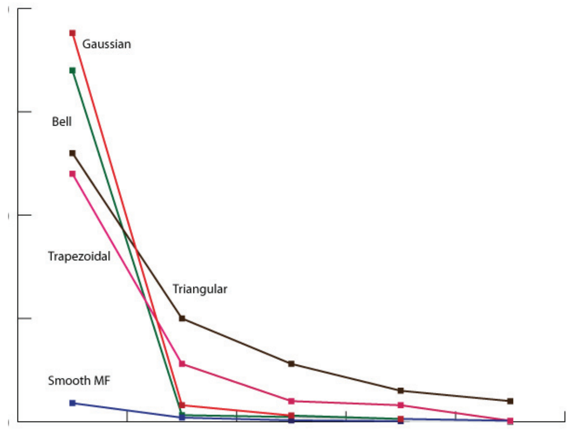

Figure 4. Minimization of error function of neural network wirh triangular and trapezoidal membership functions

VI. Conclusion

For experiments of neural network with membership functions from different classes identical database of Azerbaijani hand-printed characters was used. In fig. 4 is described graphics of error function of trained network with triangular, trapezoidal, Gaussian, Bell membership functions and MF from smooth class of membership functions.

Results of offered algorithm for recognition of Azerbaijani hand-printed characters with MF from different classes were compared with existing methods [12].

Results of recognition systems with neuro-fuzzy sets with membership functions from different classes and recognition system created for Azerbaijani hand-printed texts using neural networks were described in table 1. From the table we can see that efficiency of recognition systems with neuro-fuzzy sets with Gaussian and Bell membership functions is close to efficiency of recognition system created for Azerbaijani hand-printed texts using neural networks and recognition systems with neuro-fuzzy sets with membership functions from smooth class of membership functions is higher.

TABLE 1. COMPARING RESULTS OF RECOGNITION SYSTEMS

No	Recognition Systems	Recognition Results
1.	Recognition system with neural networks	91,1%
2.	Recognition system with neuro-fuzzy sets using triangular MF	85,5%
3.	Recognition system with neuro-fuzzy sets using trapezoidal MF	87,3%
4.	Recognition system with neuro-fuzzy sets using Gaussian MF	90,9%
5.	Recognition system with neuro-fuzzy sets using Bell MF	91,8%
6.	Recognition system with neuro-fuzzy sets using smooth MF	94,5%

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