Mathematical Computer Games Based on Modular Arithmetic

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***Abstract*— In this paper, computer aided games, which motivate and bring out the mathematical abilities of students, have been studied by inspiring "Bachet's Game". The winning strategies of the games are based on modular arithmetic. Winning algorithms and mathematical models are included in the paper.**

Levels of the logical and math games which have been proved to be beneficial for students in order to gain ability of appropriate selection and application of analytical technics and strategies in terms of problems could be applied in compliance with age group of students and be permanent during the academic life of the student as a result of its character.

***Keywords*— Computer game, Math game, Mind game, Modular arithmetic, Winning strategies, Logical strategy game, Computer game programming.**

I. Introduction

Inspiring "Bachet's Game", some games have been produced in a similar way. Modular arithmetic has been used for searching mathematical structures of the games [1]. In order to develop winning strategies and algorithms, it is benefited from techniques of Artificial Intelligence and Theory of Algorithms [2,3]. The winning strategies of the games are based on modular arithmetic. Mathematical models of the games have been examined in detail, and winning strategies (algorithms) have been proposed for different cases. Levels of the logical and math games which have been proved to be beneficial for students in order to gain ability of appropriate selection and application of analytical technics and strategies in terms of problems could be applied in compliance with age group of students and be permanent during the academic life of the student as a result of its character.

II. Types of Bachet's Game

Bachet's Game is a mathematical game whose winning strategy is based on modular arithmetic. The game is played in a competitive environment. There are N items on a table, and a player makes a move by removing at most S items from the table. The player who removes the last item loses the game. The game is diversified by values of N and S [4].

A. "Fifteen Items" Game

Fifteen items are given. Players can remove one or two items in turn. The player who removes the last item loses the game.

Let us arrange the items in groups as below.

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Figure 1. Grouping of the items

As it is seen in "Fig. 1", in order that the first player wins the game, he should remove two items in his first turn. Then, he should make the number of items, that is removed by the opponent, up to three in the latter turns. In other words, if his opponent removes one item, he should remove two items, or if the opponent removes two items, he should remove one item. The second player must remove the last item, and lose the game in this way.

1) Generalization of "Fifteen Items" Game

Given n items. Two players can remove s items ($1 \le s \le m,\ \ m < n$) respectively. The player who removes the last item loses.

Let us arrange the items in groups in a similar way as "Fifteen items" game.

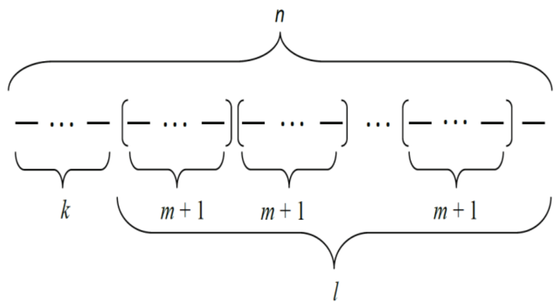

Figure 2. Grouping of items in the general game

As it is seen in "Fig. 2", the first player should remove k items in his first turn in order to win. In latter turns, the first player should remove ($m+1-s$) items when the second player removes s items ($s \le m$). Therefore, the second player has to remove the last item and loses.

We can see that this strategy is also valid for "fifteen items" game. Since $n = 15,\ \ m = 2$ in the game, as in (1), k = 2 is obtained.

$$k = (n-1)\left\lfloor \frac{n-1}{m+1} \right\rfloor = 14 - \left\lfloor \frac{14}{3} \right\rfloor \cdot 3 = 14 - 4 \cdot 3 = 2 \tag{1}$$

2) Another Version of the Game

In the above "n : m"game, the player who removes the last item loses the game. Let us consider the game with a condition that if the player removes the last item, he wins. In other words, given n items, two players can remove s items ($1 \le s \le m,\ \ m < n$) respectively. The player who removes the last item wins.

As it seen in "Fig. 3", we arrange the items in groups.

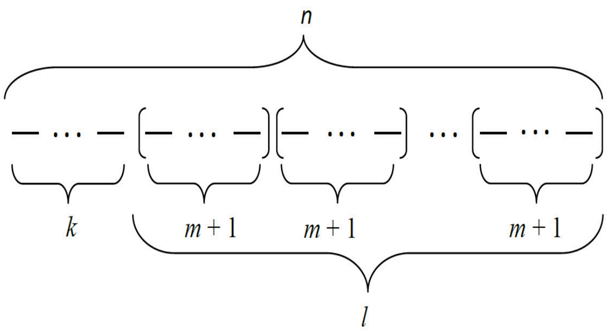

Figure 3. Grouping of items in the new version

The first player should remove k items, $k \le m$, by result (3) in order to win. In latter turns, the first player should remove ($m+1-s$) items when the second player removes s items ($s \le m$). Therefore, there remain ($m+1$) items on the table at step ($l+1$) according to value l calculated by equation (2). The second player can remove at most m items; so the first player wins the game by removing the rest of them.

$$l = \left\lfloor \frac{n-1}{m+1} \right\rfloor \tag{2}$$

$$k \equiv n \ (\mathrm{mod}\ (m+1)\ ) \tag{3}$$

B. "Hundred : ten" Game

The game is played with two people. The first player tells a number between 1 and 10. The second player tells a new number by adding at least 1 at most 10 to the previous number. The game continues like this; and the player who tells the number 100 wins the game.

The strategies mentioned before can be applied for this game as well.

Here, $n = 100,\ \ m = 10$ and $m+1 = 11$.

100-11=89 (4)

89-11=78 (5)

78-11=67 (6)

67-11=56 (7)

56-11=45 (8)

45-11=34 (9)

34-11=23 (10)

23-11=12 (11)

12-11=1 (12)

The first player should tell the number 1 (one) to win this game. Then, he should make the number s, that is told by the opponent, ($s \le 10$), up to 11 in the latter turns. In this way, he tells 12, 23, 34, 45, 56, 67, 78, 89 and 100 at last. Therefore, the number 100 is told by the first player.

1) Generalization of "Hundred : ten" Game

The game is played with two people. The first player tells a number between 1 and m. The second player tells a new number by adding at least 1 at most m to the previous number. The game continues like this; and the player who tells the number n wins the game.

We use the formula (13) and find the number k.

$$k = n - \left\lfloor \frac{n}{m+1} \right\rfloor (m+1) \tag{13}$$

The first player should tell the number k at the beginning. Then, he should make the number s, that is told by the opponent, ($s \le m$), up to m+1 in the latter turns. In other words, he tells ($m+1-s$) at each turn, so the number n is told by the first player.

III. Designing Computer Programs For Bachet's Game

Inspiring so-called "Bachet's Game", some games have been produced in a similar way, then computer programs have been proposed by making different generalizations of the games [4]. Two of the produced programs are run by being loaded to computer and the other one is played on the Internet. C# has been used for the first two of the games (ITEM and NUMBER), and the other game (MODAR) has been designed in JavaScript and Html.

A. MODAR Game

MODAR game takes its name from the union of the first letters of the words "**Mod**ular" and "**Ar**ithmetics". This game has been designed for ten items by JavaScript and Html with the name MODAR, and it can be played on *http://www.matcolik.com/oyun/nim/* [5].

MODAR game has been generated by using web programming languages html and javascript. The game can be played against the computer or a human opponent. When playing against the computer, there are three levels of the game. The computer uses all winning strategies at the most difficult level. There are ten balls in the game. Choosing the first player the game starts. The players can remove one or two balls respectively. The player who removes the last ball wins the game.

B. Programming Generalized Fifteen Items Game by C#

There are input units where we can change the total number of objects and the number of items that can be removed by players in our program. If desired, the computer or we can start the game. The starting view of the program is like in "Fig. 4".

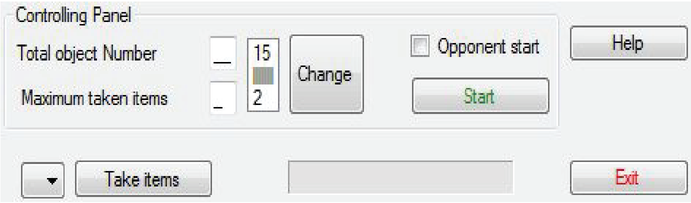

Figure 4. The starting view of the program

When pressing the Start button, there will be definite number of squares on the screen as in "Fig. 5" that we have determined at the beginning. At each step, we also determine that how many items we remove, then this number of squares will disappear from the screen.

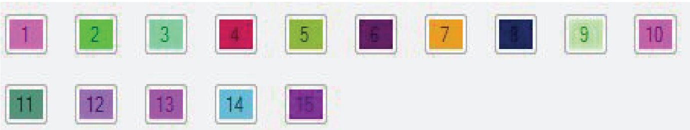

Figure 5. The view of items

We can reach the information about how the game is played by pressing the Help button.

If we lose the game a message box will be appeared on the screen.

The computer plays according to the winning strategies when playing against the computer.

C. Programming Generalized "Hundred:Ten" Game by C#

In the program, we determine the target number and the increasing number ourselves by writing them into "New Target" and "New Increasing" boxes.

The opponent (computer) or we can start the game. The starting view of the program is like in "Fig. 6".

We can reach the information about how the game is played by pressing the "Help" button.

When we press the "Start" button, the game starts.

We write the number of items, that we will remove, into "Increasing" box, the game continues like this.

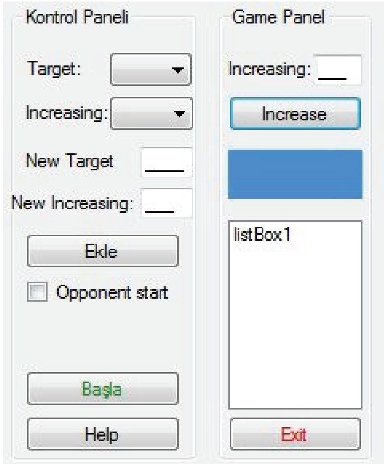

Figure 6. The starting view of the program

IV. Conclusion

A combinatoric game has been considered in this paper, moreover its generalizations and different versions have been analysed by using modular arithmetic. New logical games, that are not at the market, have been generated, solution algorithms have been proposed and computer programs have been designed. It is aimed that these programs will provide students to use mathematics, mind and game strategies by several math games.

It has been seen that generating different new games is possible by changing the conditions in mentioned games. Some strategies have been proposed and algorithms have been designed for solutions. Furthermore, the algorithm of a game that runs at an interactive web environment has been written by javascript language.

It has been shown that how mathematics can be used effectively in games and a new activity has been added to activities that can make students like mathematics.

The computer programs can be used for education in schools. Mind games, whose main purpose is to help bringing up individuals who think, questionize, reason and analyse, provide students to meet analytic thinking techniques and to improve these skills when spending enjoyable time.

References

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