SOFTWARE FOR THE SOLUTION OF SOME PRACTICAL OPTIMIZATION PROBLEMS

Knyaz Mammadov[1], Sagif Huseynov[2], Mehman Yusifov[3], Irada Bakhshaliyeva[4]

Cybernetics Institute of ANAS, Baku, Azerbaijan
[1]*mamedov_knyaz@yahoo.com*, [2]*H_Saqif@box.az*, [4]*badamdar1@rambler.ru*

I. The optimization model for the solution to the problem of replacement and connection of the objects [1].

Problem statement: How to replace m objects in n settlements that each $j-th$ ($j=\overline{1,n}$) object be connected only with one $i-th$ object, $i-th$ object be connected with maximum p_i - ($i=\overline{1,m}$) settlements, all objects be connected and the expenditures be minimal.

Let's make the following denotations:

c_{ij} is a connection expenditure between $i-th$ and $j-th$ settlements, $i=\overline{1,m}$.

a_i is a construction expenditure of the object in the $i-th$ ($i=\overline{1,m}$) point.

l_{ik} is a connection expenditure between $i-th$ and $j-th$ objects, $i=\overline{1,m}$), $k=\overline{1,m}$.

P_i is a power of the $i-th$, $i=\overline{1,m}$ object, i.e. the maximal number of the objects that have connection with this object.

$$x_{ij}=\begin{cases}1, \text{ if } i-th \text{ object has connection with } j-th \text{ settlement}\\ 0, \text{ otherwise, } (i=\overline{1,m};\ j=\overline{1,n}).\end{cases}$$

$$y_i=\begin{cases}1, \text{ if an object is replaced in the } i-th \text{ point,}\\ 0, \text{ otherwise } (i=\overline{1,m}).\end{cases}$$

The total expenditure for the establishment of the objects is $\sum_{i=1}^{m} a_i y_i$, for the connection between objects and settlements is $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$. If each point has connection with only one object then it may be written as $\sum_{i=1}^{m} x_{ij}=1$, $j=\overline{1,n}$. As each object has connection with maximum p_i points, the following inequalities must be satisfied $\sum_{j=1}^{n}x_{ij}\le p_i y_i$, ($i=\overline{1,m}$).

Considering these the mathematical model for the optimal replacement may be written as follows

$$\sum_{i=1}^{m}\ \sum_{j=1}^{n} c_{ij}x_{ij}+\sum_{i=1}^{m} a_i y_i+\sum_{i-1}^{m}\ \sum_{k=1}^{m} l_{ik}y_i y_k \to \min \tag{1}$$

$$\sum_{i=1}^{m} x_{ij}=1, \quad j=\overline{1,n} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} \le p_i \cdot y_i\ , \quad i = \overline{1,m} \tag{3}$$

$$x_{ij} = 1 \vee 0\ , \qquad y_i = 1 \vee 0\ , \quad i = \overline{1,m}\ , \quad j = \overline{1,n}\ . \tag{4}$$

The program for the realization of the method for the solution of the problem (1)-(4) offered by the authors in [1] is developed in Turbo Pascal. This program is applicable to the solution of some practical problems. For instance, solution of the $m \times n = 100 \times 100$ dimensional problem by PC Pentium IV (3,2GHz) takes approximately 2 minutes.

Note. The problem of finding the optimal replacement of the platforms and their connection during the exploitation of the layers by tangent wells may be reduced the model (1)-(4). In this case as an object may be taken the platforms and as the points – the number of the wells.

II. Partially Bul programming problem with one restriction.

The mathematical model of the problem is

$$\sum_{j=1}^{N} c_j x_j \to \max \tag{5}$$

$$\sum_{j=1}^{N} a_j x_j \le b\ , \tag{6}$$

$$x_j \in \{0,1\}, \quad j = 1,...,n, \quad n \le N \tag{7}$$

$$0 \le x_j \le 1, \quad j = n+1,...,N \tag{8}$$

Here c_j, a_j, $\left(j = \overline{1,N}\right)$ and b are positive integers. To find the optimal solution of the problem (5)-(8) the authors developed new effective method in [2]. The program of the method is developed in Turbo Pascal and allows one to solve the problem with 2000 variables. It takes approximately 5-8 minutes by PC Pentium IV (3.2 GHz).

III. Integer programming problem with one restriction.

The mathematical of the problem may be written as follows

$$\sum_{j=1}^{n} c_j x_j \to \max \tag{9}$$

$$\sum_{j=1}^{n} a_j x_j \le b \tag{10}$$

$$0 \le x_j \le d_j \quad \text{and } x_j \ \left(j = \overline{1,n}\right) \text{ is integer}\ . \tag{11}$$

Here we suppose that $c_j > 0,\ \ a_j > 0,\ \ (j = \overline{1,n})$ and $b > 0$ are arbitrary numbers. Let's give an economical interpretation to the problem (9)-(11).

Suppose that, some industrial unit has to produce n number products. The expenditure to produce the $j-th$ $\left(j = \overline{1,n}\right)$ is a_j and income from its sale is c_j ($j = \overline{1,n}$). The problem is: how to distribute the production that the total expenditures be no more than b and income be maximal. Mathematically this problem is described by (9)-(11). Here x_j is the amount of the $j-th,\ j = \overline{1,n}$ production. Note that if $d_j = 1$ $\left(j = \overline{1,n}\right)$ then (9)-(11) turns to the problem of choice of the optimal group from n objects by the total production expenditures b . To solve the problem (9)-(11) an effective method is proposed in [3]. The program software is developed in Delphi 7. The solution of 10.000 variable practical problem by this method takes approximately 2-3 minutes by PC Pentium IV (3.2 GHz).

IV. Integer linear programming problem.

The mathematical model of the problem is as follows

$$\sum_{j=1}^{n} c_j x_j \to \max \tag{12}$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \ , \ \left(i = \overline{1,m}\right), \tag{13}$$

$$0 \le x_j \le d_j ; \ x_j \ \left(j = \overline{1,n}\right) \text{ is integer.} \tag{14}$$

Here is assumed that $c_j > 0, \ a_{ij} \ge 0, \ b_i > 0, \ d_j > 0 \ \left(j = \overline{1,n}; \ i = \overline{1,m}\right)$ are given number. Let's give some economical interpretation to the problem (12)-(14).

Suppose that, some industrial unit has to produce n number products. To do this it will be used m number recourses. Denote by b_i $\left(i = \overline{1,m}\right)$ the volume of the $i-th$ recourse. Suppose that the cost of the $j-th$ $\left(j = \overline{1,n}\right)$ product is c_j $\left(j = \overline{1,n}\right)$. The volume of the $i-th$ recourse used for the production of the unit $j-th$ product denote by a_{ij}. If denote by x_j the number of the produced $j-th$ product, then d_j $\left(j = \overline{1,n}\right)$ will describe its maximal volume. Thus the problem mathematically is described by (12)-(14).

The effective method is proposed by authors in [4] to find the suboptimal solution of this problem, its sequential approximation, and error estimation.

The program software is developed in Delphi 7. It allows one to solve enough large dimensional (for instance $m \times n = 50 \times 5000$) problems by PC Pentium IV (3.2 GHz) in 5-7 minutes.

Also a Turbo Pascal software of the potentials' method is developed for the optimal solution of the known transport problem.

Note that the above mentioned program software are developed by author in the laboratory N14 "Discrete optimization models and methods" of the Institute of Cybernetics of ANAS and allows to solve any practical problems.

References

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