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ESTIMATION ECOLOGICAL SUSTAINABILITY INDEX ON THE BASE OF FUZZY LOGIC

Ramin Rzayev[1], Samir Pur Riza[2], Malahat Murtuzayeva[3]

Cybernetics Institute of ANAS, Baku, Azerbaijan
[1]*raminrza.@yahoo.com*, [2]*pqs@rambler.ru*, [3]*malaxat55@rambler.ru*

For calculate of the Ecological Sustainability Index are using by values of the many indicators, where the main are the indicators sustainability of air quality, water quality, land quality and quality of biodiversity of the considered region. And indicators of air and water qualities dominate over the others owing to the objective reasons. In other words, from the estimations levels of sustainability of this indicator formalize the general estimation of an "ecology sustainability" category. Obviously, that the estimation of ecology sustainability are the multi-criterial procedures meaning application of a composite rule of aggregate estimation of each region on the basis of accessible information about the qualities of air, water, land and biodiversity. As a rule, these data are semistructured and that is why for their adequate represent more advisable use fuzzy numbers.

For assessment the indicators of sustainability of air quality, water quality, land quality and biodiversity we choose the five level of composition: u_1 – low; u_2 – below the average; u_3 – average; u_4 – above the average; u_5- high. Better to say, under the set $C=(u_1,u_2,u_3,u_4,u_5)$ we will understand set of signs on which levels of sustainability of the indicators are classified. Then, suppose criteria of assessment by fuzzy sets, the procedure of multi-criterial assessment of ecological sustainability we realize with use of sufficient set of fuzzy implicate rules in form as "If…, then…", and on that basis we establish for it the corresponding gradation scale. Further, on the given scale we realize a quantities estimation of ecological sustainability of region.

For establish levels of the ecological sustainability we will used by next set of consistent implicate rules, which take into account indicators of air and water quality over the indicators of land quality and biodiversity:

e_1: if air quality sustainable comprehensible and water quality are standard, then the level of ecological sustainability is satisfactory (meets the minimum standard requirements);

e_2: if in addition to the above-stated requirements biodiversity is sustainable does not vary, then the level of ecological sustainability is more than satisfactory;

e_3: if additionally to conditions e_2 the land quality saves sustainable state, then the level of ecological sustainability is perfect;

e_4: if in considered region take place stipulated in e_3 except a biodiversity, then the level of ecological sustainability is very satisfying;

e_5: if the air quality sustainable comprehensible, water quality standard, land quality saves sustainable state, however biodiversity is not sustainable, then the level of ecological sustainability of the region nevertheless will be satisfactory;

e_6: if the air quality is not sustainable and land quality does not save sustainable state, then the level of ecological sustainability of region unsatisfactory.

The analysis of the resulted statements allow to reveal consider to offered model four exogenous criteria using to the estimation of ecology sustainability: X_1 – sustainable quality of air consist; X_2 – sustainable quality of water consist; X_3 – sustainable quality of land consist; X_4– sustainable of biodiversity, and the endogen sign Y – sufficiency of the level of ecological sustainability. Then having defined the considered valued (fuzzy terms) of linguistic variables X_i (i=1÷4) and Y, on the basis of the resulted statements we will construct fuzzy implicative rules in the form:

e_1: if X_1=SUSTAINABLE COMPREHENSIVE and X_2=SUSTAINABLE STANDARD, then Y=SATISFACTORY;

e_2: if X_1=SUSTAINABLE COMPREHENSIVE and X_2=SUSTAINABLE STANDARD and X_4=SUSTAINABLE INVARIABLE, then Y=MORE THAN SATISFACTORY;

e_3: «if X_1=SUSTAINABLE COMPREHENSIVE and X_2=SUSTAINABLE STANDARD and X_3=SUSTAINABLE DOES NOT VARY (CHANGE) and X_4=SUSTAINABLE CONSTANT, then Y=IRREPROACHABLE;
e_4: if X_1=SUSTAINABLE COMPREHENSIVE and X_2=SUSTAINABLE STANDARD and X_3=SUSTAINABLE DOES NOT VARY (CHANGE), then Y=VERY SATISFACTORY;
e_5: if X_1=SUSTAINABLE COMPREHENSIVE and X_2=SUSTAINABLE STANDARD and X_3=SUSTAINABLE DOES NOT VARY(CHANGE) and X_4= UNSUSTAINABLE, then Y=SATISFACTORY;
e_6: if X_1=UNSUSTAINABLE and X_3=WORSENS, then Y=UNSATISFACTORY.

Further, linguistic variable Y we will set on discrete set J = (0;0.1;0.2;..;1). Then using in implicative rules its values – fuzzy terms we can write with the help of the next membership functions [1]: $\widetilde{S}$=COMPREHENSIVE ($\mu_{\widetilde{S}}(x) = x$, $x \in J$); $M\widetilde{S}$=MORE THAN COMPREHENSIVE ($\mu_{M\widetilde{S}}(x) = \sqrt{x}, x \in J$); $\widetilde{P}$=IRREPROACHIBLE ($\mu_{\widetilde{P}}(x) = \begin{cases} 1, & x = 1, \\ 0, & x < 1, \end{cases}$ $x \in J$); $V\widetilde{S}$=VERY COMPREHENSIVE as: $\mu_{V\widetilde{S}}(x) = x^2$, $x \in J$; $U\widetilde{S}$=UNCOMPREHENSIVE ($\mu_{U\widetilde{S}}(x) = 1 - x$, $x \in J$). Fuzzifications of terms in the left part accepted rules we will realize with the help of Gaussian membership functions $\mu_{\widetilde{E}}(u) = \exp\{-(u-100)^2 / \sigma_k^2\}$ (k=1÷5) restoring fuzzy sets on base vector (0, 25, 50, 75, 100) (fig.1) and values for σ_k steal up from degree of important of an indicator.

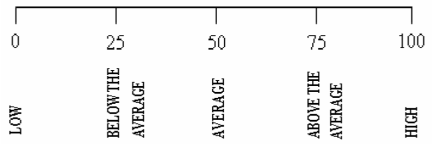

Fig.1 Components of the base vector

Thus, criteria estimation of sustainability indicators we will define in the form of following fuzzy sets:

- SUSTAINABLE COMPREHENSIVE: $\widetilde{A} = \frac{0.000015}{u_1} + \frac{0.00193}{u_2} + \frac{0.0622}{u_3} + \frac{0.4994}{u_4} + \frac{1}{u_5}$;
- SUSTAINABLE STANDARD: $\widetilde{B} = \frac{0.000015}{u_1} + \frac{0.00193}{u_2} + \frac{0.0622}{u_3} + \frac{0.4994}{u_4} + \frac{1}{u_5}$;
- SUSTAINABLE DOES NOT VARY: $\widetilde{C} = \frac{0.000285}{u_1} + \frac{0.010134}{u_2} + \frac{0.1299}{u_3} + \frac{0.6}{u_4} + \frac{1}{u_5}$;
- SUSTAINABLE CONSTANT: $\widetilde{D} = \frac{0.00193}{u_1} + \frac{0.02973}{u_2} + \frac{0.2096}{u_3} + \frac{0.676634}{u_4} + \frac{1}{u_5}$;

Then with the account of these formalisms we formalize fuzzy rules as:
e_1: If X_1=$\widetilde{A}$ and X_2=$\widetilde{B}$, then Y=$\widetilde{S}$;
e_2: If X_1=$\widetilde{A}$ and X_2=$\widetilde{B}$ and X_4=$\widetilde{D}$, then Y=$\widetilde{M\widetilde{S}}$;
e_3: If X_1=$\widetilde{A}$ and X_2=$\widetilde{B}$ and X_3=$\widetilde{C}$ and X_4=$\widetilde{D}$, then Y=$\widetilde{P}$;
e_4: If X_1=$\widetilde{A}$ and X_2=$\widetilde{B}$ and X_3=$\widetilde{C}$, then Y=$V\widetilde{S}$;
e_5: If X_1=$\widetilde{A}$ and X_2=$\widetilde{B}$ and X_3=$\widetilde{C}$ and X_4=$\neg\widetilde{D}$, then Y=$\widetilde{S}$;
e_6: If X_1=$\neg\widetilde{A}$ and X_3=$\neg\widetilde{C}$, then Y=$U\widetilde{S}$.

Further, having calculated membership functions $\mu_{\widetilde{M}_i}(u)$ (i=1÷5) for left parts of these rules we will obtain in results:

e_1: «If $X=\widetilde{M}_1$, then $Y=\widetilde{S}$; e_2: «If $X=\widetilde{M}_2$, then $Y=M\widetilde{S}$ »; e_3: «If $X=\widetilde{M}_3$, then $Y=\widetilde{P}$ »; e_4: «If $X=\widetilde{M}_4$, then $Y=V\widetilde{S}$ »; e_5: «If $X=\widetilde{M}_5$, then $Y=\widetilde{S}$ »; e_6: «If $X=\widetilde{M}_6$, then $Y=U\widetilde{S}$ ».

For transformation of these rules we will take advantage Lucasevich implication. Then for each pair $(u,j)\in U\times Y$ на $U\times Y$ we will construct the following fuzzy relations R_1, R_2, …, R_6, the crossing of which give in results the required general functional decision:

$R=$

	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
u_1	1	0,900285	0,800285	0,700285	0,600285	0,500285	0,400285	0,300285	0,200285	0,100285	0,000285
u_2	0,999877	0,910134	0,810134	0,710134	0,610134	0,510134	0,410134	0,310134	0,210134	0,110134	0,010134
u_3	0,981684	0,981684	0,929923	0,829923	0,729923	0,629923	0,529923	0,429923	0,329923	0,229923	0,129923
u_4	0,632121	0,632121	0,632121	0,632121	0,632121	0,632121	0,632121	0,632121	0,632121	0,632121	0,600373
u_5	0	0	0	0	0	0	0	0	0	0	1

For calculate of sufficiency ecology sustainable of each of five regions we will applicate rule of compositional output in fuzzy environment: $\widetilde{E}_k=\widetilde{G}_k\circ R,$ where $\widetilde{E}_k$ – sufficiency degree of ecological sustainability of k-th region, $\widetilde{G}_k$ – mapping of *k*-th region in form of fuzzy subset on *U*. Then we have [1]: $\mu_{\widetilde{E}_k}(j)=\max_u\big(\min(\mu_{\widetilde{G}_k}(u),\mu_R(u))\big),$ where $\mu_{\widetilde{G}_k}(u)=\begin{cases}0, u\neq u_k;\\ 1, u=u_k.\end{cases}$ It follows from here, that $\mu_{\widetilde{E}_k}(j)=\mu_R(u_k,j),$ i.e. $\widetilde{E}_k$ is a k-th line of matrix *R*.

Now for first level we have value in fuzzy sets form:

$$\widetilde{E}_1=\frac{1}{0}+\frac{0.900285}{0.1}+\frac{0.800285}{0.2}+\frac{0.700285}{0.3}+\frac{0.600285}{0.4}+\frac{0.500285}{0.5}+$$
$$+\frac{0.400285}{0.6}+\frac{0.300285}{0.7}+\frac{0.200285}{0.8}+\frac{0.100285}{0.9}+\frac{0.000285}{1.0}.$$

Let's calculate it sets level $E_{j\alpha}$ and corresponding capacities $M(E_{j\alpha})$ by formulas $M(E_{j\alpha})=\sum_{i=1}^{n}\frac{x_i}{n}$:

- for $0<\alpha<0.000285$: $\Delta\alpha=0.000285$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.6;\ 0.7;\ 0.8;\ 0.9;\ 1\}$, $M(E_{1\alpha})=0.50$;
- for $0.000285<\alpha<0.100285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.6;\ 0.7;\ 0.8;\ 0.9\}$, $M(E_{1\alpha})=0.45$;
- for $0.100285<\alpha<0.200285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.6;\ 0.7;\ 0.8\}$, $M(E_{1\alpha})=0.4$;
- for $0.200285<\alpha<0.300285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.6;\ 0.7\}$, $M(E_{1\alpha})=0.35$;
- for $0.300285<\alpha<0.400285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.6\}$, $M(E_{1\alpha})=0.3$;
- for $0.400285<\alpha<0.500285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5\}$, $M(E_{1\alpha})=0.25$;
- for $0.500285<\alpha<0.600285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3;\ 0.4\}$, $M(E_{1\alpha})=0.20$;
- for $0.600285<\alpha<0.700285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2;\ 0.3\}$, $M(E_{1\alpha})=0.15$;
- for $0.700285<\alpha<0.800285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1;\ 0.2\}$, $M(E_{1\alpha})=0.10$;
- for $0.800285<\alpha<0.900285$: $\Delta\alpha=0.1$, $E_{1\alpha}=\{0;\ 0.1\}$, $M(E_{1\alpha})=0.05$;
- for $0.900285<\alpha<1$: $\Delta\alpha=0.099715$, $E_{1\alpha}=\{0\}$, $M(E_{1\alpha})=0$;

Further, we find a dot estimation of sufficiency of ecological sustainability first region as:

$$F(\widetilde{E}_1)=\frac{1}{1}\int_0^1 M(E_{1\alpha})d\alpha=0.225142.$$

By similar actions it is establish dot estimation for ecology sustainability on other levels: for level u_2 $F(\widetilde{E}_2)=0.230096$; for u_3 $F(\widetilde{E}_3)=0.292914$; for u_4 $F(\widetilde{E}_4)=0.497489$; for u_5 $F(\widetilde{E}_5)=1$. As region with most satisfactory indicator of ecological sustainability we choose region having greatest dot estimation. It is region u_5 in resulted example. On the second place – u_4, on the third – u_2, on the fourth – u_3 and, at last, on the fifth – u_1. Thus, in accepted supposes the final scales for estimation of ecology sustainability will look as it is shown on the fig.2.

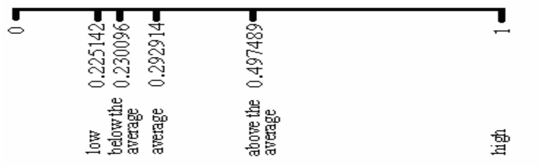

Fig.2 The scale for assessment of ecology sustainability

After established the proved scale for estimation of ecology sustainability we will obtain a dot estimation for indicators of sustainability of air quality, water quality, land quality and biodiversity, obtained in work [2] in corresponding values form on hundred-scores system calculator: x_1=37.5; x_2=62.5; x_3=50; x_4=62.5. For obtain final estimation we will construct following trivial implicative rules:

- If sustainability of air quality (AQ) is low and sustainability of water quality (WQ) is low and sustainability of land quality (LQ) is low and sustainability of biodiversity (NB) is low, then ecology sustainability (ES) of region is low;
- If sustainability of air quality (AQ) below the average and sustainability of water quality (WQ) below the average and sustainability of land quality (LQ) below the average and sustainability of biodiversity (NB) below the average, then ecology sustainability (ES) of region below the average;
- If sustainability of air quality (AQ) average and sustainability of water quality (WQ) average and sustainability of land quality (LQ) average and sustainability of biodiversity (NB) average, then ecology sustainability (ES) of region is an average;
- If sustainability of air quality (AQ) above the average and sustainability of water quality (WQ) above the average and sustainability of land quality (LQ) above the average and sustainability of biodiversity (NB) above the average, then ecology sustainability (ES) of region are above the average;
- If sustainability of air quality (AQ) is high and sustainability of water quality (WQ) is high and sustainability of land quality (LQ) is high and sustainability of biodiversity (NB) is high, then ecology sustainability (ES) of region is high.

Having realize these rules in MATLAB/Fuzzy Sets Toolbox (fig.3), on the basic of indicators x_1, x_2, x_3, x_4 will obtain final estimation of ecology sustainability at a rate of 0.295. According to the accepted scale this value is close to average level of ecology sustainability

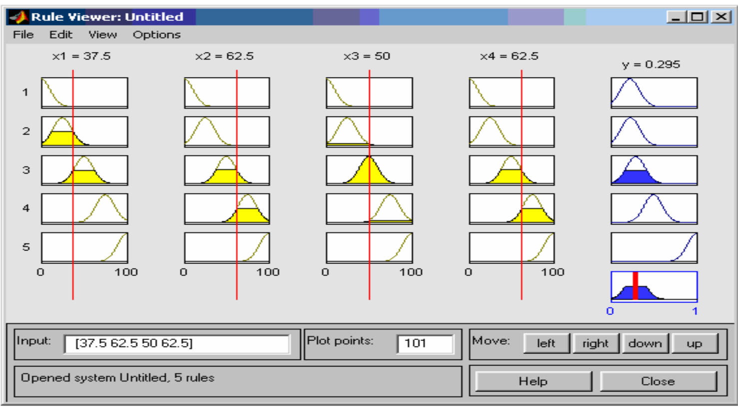

Fig.3 Assessment of ecology sustainability in MATLAB notation.

References

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