EVENTOLOGICAL "CHESS" MODEL OF BEHAVIOR OF MARKET AGENTS

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An eventological model (E-model) based on the eventological principle of duality of events [1] and on the eventological approach of modeling of commodity market [2, 3, 4] was built. The model is called the "Chess" model and describes the joint economic and psychological behavior of market participants. This E-model extends the classic understand about the marketable and the psychological conflicts of demand and supply, contemplation and action.

Mathematical eventology [1] is the new trend in the probabilistic theory, it studies events and their interactions. The eventology offers to switch from a numeric level of random events to the set level of analysis. The eventology studies random events and their distributions first of all but never forgets also about random elements and their distributions generated by these random events.

We offer a "Chess" market E-model based both on the eventological principle of duality of events and on the approach to modeling the goods market as event's market. This E-model describes the joint economic and psychological behavior of market participants and extends the classical idea of the marketable and psychological conflicts of event-demand, event-supply, event-contemplation and event-activity.

1. Goods market

The commodity market is one of the most striking examples for demonstration of diversity structures of dependencies of events. From the eventological point of view any commodity market can be regarded as event's market: the market of events-demand and events-supply of this product. However these market events are making by market agents (consumers/producers). In turn a psychological and a market mechanisms are always functioning "inside'' of each market agent (consumer/producer). These market mechanisms define the individual behavior of each market participant in according to his market and psychological tastes and preferences. In other words, it is natural to assume that every market participant may be in one of a set of permissible market and psychological states.

1.1. E-model of a market agent

Let consider under the probabilistic space an E-model of one participant of the commodity market (market agent) as a set of events-terraces

$$\left\{\mathrm{ter}(X_B^L), (X, B, L) \in 2^{\mathfrak{X}} \times 2^{\mathfrak{B}} \times 2^{\mathfrak{L}}\right\} \subseteq \mathcal{F},$$

numbered by elements of the direct product

$$2^{\mathfrak{X}} \times 2^{\mathfrak{B}} \times 2^{\mathfrak{L}} = \{(X, B, L), X \subseteq \mathfrak{X}, B \subseteq \mathfrak{B}, L \subseteq \mathfrak{L}\},$$

here $\mathfrak{X}$ is a set of goods, $\mathfrak{L} = \{\downarrow, \uparrow\}$ is a set of economic states of the given market agent, ($\downarrow$ is a demand, $\uparrow$ is a supply), $\mathfrak{B} = \{a, c\}$ is a set of psychological states of the given market agent (a is an activity, c is a contemplation). Formulae for events□terraces:

$$\mathrm{ter}(X_B^L) = \bigcap_{x \in X} x_B^L \bigcap_{x \in X^c} (x_B^L)^c, \quad X \subseteq \mathfrak{X},$$

here x_B^L is an event that occurs when a market agent is in the subsets of economic and psychological states $L \subseteq \mathfrak{L}$ and $B \subseteq \mathfrak{B}$ respectively in relation to the product $x \in \mathfrak{X}$; and for fixed subset of psychological $B \in \mathfrak{B}$ and economic $L \in \mathfrak{L}$ states of agent and for fixed subset of goods $X \subseteq \mathfrak{X}$:

$$X_B^L = X \times B \times L = \{(x, B, L), x \in X\}.$$

In particular $x_B^{\{\downarrow\}}$ is an event which occurs when the goods are *asked* ($L = \{\downarrow\}$) by a market agent in a subset of psychological states $B \in \mathfrak{B}$ and $x_B^{\{\uparrow\}}$ is an event which occurs when the goods are *supplied* ($L = \{\uparrow\}$) by a market agent in a subset of psychological states $B \in \mathfrak{B}$.
Thus subsets of economic and psychological states of the market agent are associated with one product $x \in \mathfrak{X}$ define the Kolmogorov events:

- contemplation of demand of goods $x_{\{c\}}^{\{\downarrow\}}$,
- creating of demand of goods $x_{\{a\}}^{\{\downarrow\}}$,
- contemplation of supply of goods $x_{\{c\}}^{\{\uparrow\}}$,
- creating of supply of goods $x_{\{a\}}^{\{\uparrow\}}$.

In particular the event-terrace $\mathrm{ter}(X_B^{\{\downarrow\}})$ is corresponding to the element of the direct product

$$X_B^{\{\downarrow\}} = X \times B \times \{\downarrow\} = \{(x, B, \{\downarrow\}), x \in X\}$$

when $L = \{\downarrow\}$. This event-terrace occurs when a subset $X \subseteq \mathfrak{X}$ *is asked* ($L = \{\downarrow\}$) by the market agent if he is in a subset of psychological states $B \subseteq \mathfrak{B}$.
And the event-terrace $\mathrm{ter}(X_B^{\{\uparrow\}})$ is corresponding to the element of the direct product

$$X_B^{\{\uparrow\}} = X \times B \times \{\uparrow\} = \{(x, B, \{\uparrow\}), x \in X\}$$

when $L = \{\uparrow\}$. This event-terrace occurs when a subset $X \subseteq \mathfrak{X}$ *is supplied* ($L = \{\uparrow\}$) by the market agent if he is in a subset of psychological states $B \subseteq \mathfrak{B}$.

1.2. E-model of set of market agents

Let consider a finite set of market agents $\mathfrak{M}$, each $\mu \in \mathfrak{M}$ is described by the E-model imposed above:

$$\left\{\mathrm{ter}(X_B^L), (X, B, L) \in 2^{\mathfrak{X}} \times 2^{\mathfrak{B}} \times 2^{\mathfrak{L}}\right\} \subseteq \mathcal{F}.$$

This E-model is a set of the Kolmogorov events characterizing the economic and psychological behavior of one agent in the commodity market.
The E-model

$$\left\{\bigcap_{\mu \in \mathfrak{M}} \mathrm{ter}(X_{\mu}{}_{B_\mu}^{L_\mu}), \left\{X_{\mu}{}_{B_\mu}^{L_\mu}, \mu \in \mathfrak{M}\right\} \in \left(2^{\mathfrak{X}} \times 2^{\mathfrak{B}} \times 2^{\mathfrak{L}}\right)^{\times|\mathfrak{M}|}\right\} \subseteq \mathcal{F},$$

is proposed as the E-model of behavior of set of agents in the commodity market. Here

$$\left(2^{\mathfrak{X}} \times 2^{\mathfrak{B}} \times 2^{\mathfrak{L}}\right)^{\times|\mathfrak{M}|} =$$
$$= \underbrace{\left(2^{\mathfrak{X}} \times 2^{\mathfrak{B}} \times 2^{\mathfrak{L}}\right) \times \ldots \times \left(2^{\mathfrak{X}} \times 2^{\mathfrak{B}} \times 2^{\mathfrak{L}}\right)}_{|\mathfrak{M}|}.$$

is the direct product of "triples" of sets, each of which corresponds to the E-model of a single market agent $\mu \in \mathfrak{M}$.

2. A pure consumer and a pure producer

If we'll fix an arbitrary subset of goods $X \subseteq \mathfrak{X}$ the E-model of a market agent is reduced to a set

$$\left\{\mathrm{ter}(X_B^L), \quad B \subseteq \mathfrak{B}, L \subseteq \mathfrak{L}\right\} \subseteq \mathcal{F}$$

consisting of $256 = 16 \times 16$ events-terraces. This set eventologically describes the commodity market relative to a fixed arbitrary subset of goods $X \subseteq \mathfrak{X}$.

Let consider a very special version of the E-model of the set of market agents (see the section 1.2), this model is the E-model of two market agents. Moreover let each of them can be only in one economic state $\{\downarrow\} \in L$ or $\{\uparrow\} \in L$ and in a subset of psychological states $B \subseteq \mathfrak{B}$.

If a market agent is in the economic state $\{\downarrow\} \in L$ then we call it *a pure consumer*. The pure consumer only asks or does not ask a subset of goods $X \subseteq \mathfrak{X}$ and he is described by a set $\left\{ \mathrm{ter}(X_B^{\{\downarrow\}}), \quad B \subseteq \mathfrak{B} \right\} \subseteq \mathcal{F}$, consisting from 8 events-terraces.

If the market agent is in the economic state $\{\uparrow\} \in L$ then we call it *a pure producer*. The pure producer only supply or does not supply a subset of goods $X \subseteq \mathfrak{X}$ and he is described by a set $\left\{ \mathrm{ter}(X_B^{\{\uparrow\}}), \quad B \subseteq \mathfrak{B} \right\} \subseteq \mathcal{F}$, also consists from 8 events□terraces.

The E-model of the pair of market agents represents a set

$$\left\{ \mathrm{ter}(X_{B\{\downarrow\}}^{\{\downarrow\}}) \cap \mathrm{ter}(X_{B\{\uparrow\}}^{\{\uparrow\}}),\ B^{\{\downarrow\}} \subseteq \mathfrak{B},\ B^{\{\downarrow\}} \subseteq \mathfrak{B} \right\} \subseteq \mathcal{F}, \qquad (*)$$

consisting from $64 = 8 \times 8$ intersections of events-terraces where $B^{\{\downarrow\}}$ is a subset of the psychological states of the pure consumer and $B^{\{\uparrow\}}$ is a subset of the psychological states of the pure producer.

The interaction of this pair of pure market agents splits all space of elementary market events on the 64 events-terraces:

$$\mathrm{ter}(X_{B\{\downarrow\}}^{\{\downarrow\}}) \cap \mathrm{ter}(X_{B\{\uparrow\}}^{\{\uparrow\}}),$$

which describe the event's interaction of these agents in the commodity market.

This simplified E-model conflict of two pure market agents will be called "chess" E-model of market behavior of pure consumer and pure producer (because the Vienn diagram of set of the parity intersections of events-terraces $(*)$ looks like chessboard).

3. The conditional generalized Gibbsean eventological distributions

According to the E-market model the set of every market events associated with some market agent has the generalized Gibbsean E-distribution [4] for $X \subseteq \mathfrak{X}, B \in \mathfrak{B}, L \in \mathfrak{L}$:

$$p(X_B^L) = \frac{1}{Z} \exp\{\alpha_B^L \mathcal{V}(X_B^L)\},$$

market events-terraces:

$$\mathrm{ter}(X_B^L) = \bigcap_{x \in X} x_B^L \bigcap_{x \in X^c} (x_B^L)^c.$$

The marginal distribution of market agent being only in the state of demand (the pure consumers) has the form of the generalized E-Gibbsean distribution:

$$p(X_{B^\downarrow}^{\{\downarrow\}}) = \frac{1}{Z} \exp\{\alpha_{B^\downarrow}^{\{\downarrow\}} \mathcal{V}(X_{B^\downarrow}^{\{\downarrow\}})\}, \quad B^\downarrow \subseteq \mathfrak{B},$$

and the marginal distribution of the agent being only in the state of supply (pure producer) has the generalized Gibbsean E-distribution:

$$p(X_{B^\uparrow}^{\{\uparrow\}}) = \frac{1}{Z} \exp\{\alpha_{B^\uparrow}^{\{\uparrow\}} \mathcal{V}(X_{B^\uparrow}^{\{\uparrow\}})\}, \quad B^\uparrow \subseteq \mathfrak{B}.$$

Then the joint E-distribution of two pure market agents looks for the $B^\downarrow \subseteq \mathfrak{B}, B^\uparrow \subseteq \mathfrak{B}$ in independent situations:

$$p(X_{B^\downarrow + B^\uparrow}^{\downarrow\uparrow}) = p(X_{B^\downarrow}^{\downarrow}) p(X_{B^\uparrow}^{\uparrow}).$$

And in the common situation

$$p(X_{B^\downarrow + B^\uparrow}^{\downarrow\uparrow}) = p(X_{B^\downarrow}^{\downarrow} | X_{B^\uparrow}^{\uparrow}) p(X_{B^\uparrow}^{\uparrow}),$$

where

$$p(X_{B^\downarrow}^{\downarrow} | X_{B^\uparrow}^{\uparrow}) = \frac{\mathbf{P}\left(\mathrm{ter}(X_{B^\downarrow}^{\downarrow}) \cap \mathrm{ter}(X_{B^\uparrow}^{\uparrow})\right)}{\mathbf{P}\left(\mathrm{ter}(X_{B^\uparrow}^{\uparrow})\right)}$$

is the conditional probability of event-terraces $\mathrm{ter}(X_{B^\downarrow}^{\downarrow})$ with the condition of the event-terrace $\mathrm{ter}(X_{B^\uparrow}^{\uparrow})$, or

$$p(X_{B^\downarrow + B^\uparrow}^{\downarrow\uparrow}) = p(X_{B^\uparrow}^{\uparrow} | X_{B^\downarrow}^{\downarrow}) p(X_{B^\downarrow}^{\downarrow}),$$

where

$$p(X^{\uparrow}_{B\uparrow}|X^{\downarrow}_{B\downarrow}) = \frac{\mathbf{P}\left(\mathrm{ter}(X^{\downarrow}_{B\downarrow}) \cap \mathrm{ter}(X^{\uparrow}_{B\uparrow})\right)}{\mathbf{P}\left(\mathrm{ter}(X^{\downarrow}_{B\downarrow})\right)}$$

is the conditional probability of event-terrace $\mathrm{ter}(X^{\uparrow}_{B\uparrow})$ with the condition of event-terrace $\mathrm{ter}(X^{\downarrow}_{B\downarrow})$.

4. Discussion

The "chess" E-model of the joint event's behavior of the pure consumer and the pure producer at the commodity market is offered in the paper. This model takes into account psychological states of market agents. It is based on the E-model of the goods market, which is determined by two generalized Gibbsean E-distributions. Under this approach the "chess" market E-model is described by conditional generalized Gibbsean E-distributions, which determine the relationship between economic and psychological behavior of two market agents.

The "chess" E-model is a new eventological design such as a set of events-terraces which are numerated by collections of multiple indexes, which are elements of the direct product of sets. This allows to extend the classic understanding about the market and the psychological conflicts of demand, supply, contemplation and action.

References

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