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DEFINING THE CONCEPT '*THE FUNCTIONAL STATE OF A CONTROLLED COMPLEX OBJECT BEHAVIOR*'

Nicolay Kirillov

St.Petersburg Institute for Informatics and Automation of RAS, St.Petersburg, Russia
knp@mail.ru

1. Introduction

Specifying the requirements of the mathematical models describing the behavior of artificial systems, not to mention developing and analyzing these models, is hardly possible without the concepts making the subject domain ontology[1]. One of the fundamental concepts of this ontology is *state*. Unfortunately, some ambiguities are present in the existing definitions of this concept. In particular, we would mention the incomplete coverage of some essential properties in such definitions. Moreover, some existing definitions are relying on other undefined or poorly defined concepts.

The objective of this paper is providing a comprehensive justification of the set of the essential properties comprising the concept a *functional state* of a complex controlled artificial object. One of our goals is making it applicable to all phases of its life cycle.

2. Summary of the proposed approach

Our approach boils down to the following six claims

2.1. The need for the concept of *state* emerges when we want to describe the behavior of a complex artificial object; this happens when we want to develop its mathematical model. Thus a *state* is an attribute of an abstraction of this object which is its model.

2.2. The properties of states of a complex object are determined by the properties of its model; the latter properties, in their turn, could be viewed as the requirements of the attributes of object models.

2.3. The purpose of model of a complex object determines the specifics of its attributes. As we deal with artificial objects, we assume that the models fit in the *situation control* paradigm.

2.4. Each state has its own unique identifier (name); this makes it possible to distinguish between different states used in the model description. We distinguish states by the values of the essential parameters of the complex object in the measurement time.

2.5. Each model parameter is determined by a unique set of its attributes having apparent meaning. This includes what and where is being measured, the measurement scale, sampling interval, time references, etc.

2.6. The model of the complex object behavior is a the classical Kalman's discrete state transition model (DSTM) [1]. This model could be represented by two functions β and χ:

$$\beta : X \to C \tag{1}$$

$$\chi : C \times U \times P \times T \times \mathrm{T} \to C \tag{2}$$

[1] In this text, by the term *complex object* we mean an artificial material entity used for satisfying some needs of humans.

where β is the state identification function, χ is the state transition function; X is a set of the object process parameters; C is the set of the complex object process states (this is also deemed to be the state of the controlled object); U is the set of controls; P is the set of external factors; T is the set of all time values; Τ is the set of time intervals $\tau = (t, t']$ $(t, t' \in T)$, where t is the time when the forecast for the complex object state is calculated for the current inputs to function moment χ, and t' is the time forecast time.

3. Further details

3.1. The concept *state* omnipresent in the Systems Theory is used without explicit definition. We want to fill this gap keeping in mind that *state* is an essential attribute of any complex object's DSTM. Thus we propose the following:

Definition 1: *A state is an attribute of the deterministic model of cause-effect changes in the processes pertaining some complex object. This attribute refers to given instant of time, has a unique name, semantic interpretation and could be identified by the numerical values of the set of parameters of these processes. It is meant that, the information about these parameters and the controls applied to this object, is necessary and sufficient for predicting in the discrete time the future values of these parameters with the precision that allows for uniquely identifying the name of the respective attribute*.

3.2. The descriptions of complex object behaviors used in models are taking into account some constraints that are not addressed in DSTMs. Typically, by design, the behavior of complex artificial objects could be represented as a stationary process on long lasting time intervals. While solving situational control problems for such complex objects, this allows assuming stationary and robust states of the object in the phase space when no controls and/or external disturbances from the environment are present[2].

Let T' is the time interval on that the properties of this complex object cold be assumed stationary, $T' \subset T$, and P' is some subset of the external disturbances, $P' \subset P$. Also assume that we have no way to distinguish between different elements in T' and P'. In this case, by using the DSTM for any current state $c \in C$, any control $u \in U$, and any time $\tau \in \mathrm{T}$ we can only obtain the forecast with the precision to some subset of the complex object states $\bigcup_{\forall p \in P' \forall t \in T'} \chi(c, u, p, t, \tau) = C' \subset C$.

This gives rise to developing quite different models of complex objects that will take into account all the peculiarities highlighted above. In our early work, we have demonstrated that DSTM could be used as the theoretical framework for the new generation of such models for describing the deterministic behaviors of complex objects [2].

For this purpose, for the state transition function χ in (2) we use one of the set of potentially existing *k*-monomorphic patterns. Let for i-th model this function be $\chi_i^{P'T'}$. In other words, it is the mapping

$$\chi_i^{P'T'} : C \setminus R_i \times U_i \times \mathrm{T} \to C \setminus R_i, \tag{3}$$

[2] In this context, the *environment* is comprised of all external entities, among them everything that provides resources and maintains this complex object, the external artificial and natural objects, etc.

where R_i is the equivalency relation on the set *C* of the DSTM states; $C \setminus R_i$ is the factor set whose interpretation is the set of states of the complex object in the *i*-th model; U_i is a subset of the set of controls *U*; Τ is the set of time intervals $(t_н, t_к]$. On each such time interval some controlled change in the state of the object is anticipated. The controls belong to set U_i; $t_н$ is the earliest time and $t_к$ is the latest time when the object state change may take place.

The *k*-monomorphism properties between the two mapping (1) and (3) are determined by the following condition:

$$\chi_i^{P'T'}(a, u, (t_н, t_к]) = b \Leftrightarrow$$
$$\forall p \in P' \forall t \in T' \forall c \in \gamma_i^{-1}(a) \exists \tau \subset (t + t_н, t + t_к] \{\alpha(c, u, p, t, \tau) \in \gamma_i^{-1}(b)\}, \quad (4)$$

where γ_i is the mapping $\gamma_i : C \to C \setminus R_i$ (which is natural for the equivalency R_i).

The mapping (3) must satisfy the constraint which the equilibrium property:

$$\forall a \in C \setminus R_i \, \forall \tau \in \mathrm{T} \, \chi_i^{P'T'}(a, u_\varnothing, \tau) = a, \quad (5)$$

where $u_\varnothing$ is the empty element of set U_i, whose meaning in the *i*-th model is the absence of the control input to this complex object.

The stability property of the object states immediately follows from (3); the object behavior in the i-th model is independent of any inputs in set P'.

On the common sense level, equation $\chi_i^{P'T'}(a, u, \tau) = b$ has the following meaning. If at time T' the object receives some inputs from set P' and at the time when the control $u \neq u_\varnothing$ has been applied, this object was in state *a,* then function $\chi_i^{P'T'}$ describes the change of the state of this object to *b* on the time interval τ.

The identification and common-sense interpretation of the state transition function $\chi_i^{P'T'}$ is made in values and terms of the measured parameters using this mapping:

$$\beta\gamma_i : X \to C \setminus R_i. \quad (6)$$

4. Functional model and states

In the general case, we can map function χ on the non-empty set of *k*-monomorphic patterns $\{\chi_i^{P'T'} \mid \forall i \in I\}$ satisfying (3), (4) и (5).

As we noted earlier, the model of a complex object must serve its purpose. In the situational control context, this means that there must exist some association between the set of state in the chosen model and the set of possible goal states of the object that would allow the unique identification of of each goal state in the set of all states. Mathematically this requirement could be exprssed in the following form [3]

$$R_i \subset Q, \quad (7)$$

where Q is the equivalency relation defined on the set of all states of the DSTM C so that a mutually-unique correspondence was established between the set of the equivalency classes and the set of names of the object control goal states.

All models satisfying (7) could be used for solving situational control problems in the state space of the controlled complex objects. In other words, this means that such models all have the *functionality* property[3]. On these grounds, we refer to such models as *functional models* and refer to the states used in these models as *functional states*.

Deinition 2. *A functional state is an attribute of the deterministic model of the processes pertaining the controlled complex object. This attribute:*

- *specifies the equilibrium and balanced phase of these processes on the given time interval and the conditions in that this complex object exists;*
- *has a unique name, semantical interpretation and identification in the values of the parameters of these processes; it is meant that knowing these values and the control inputs to the object is necessary and sufficient for forecasting these parameters for the future time intervals specified in this model; this forecast could be made with the precision that is sufficient for the unique identification of the name of the corresponding attribute.*

5. Conclusion

The proposed precise definitions of the widely used concepts *state* and *functional state* reflect all the essential properties of the pertaining attributes that can be found in the corresponding conceptual models of behavior of complex artificial objects. Unlike the previously know definitions, the new ones presented in (1), (2), (3), and (6) do not contain any undefined terminal concepts that need further definition.

This allows using these definitions in all phases of the life cycle of the complex object for unambiguous and correct understanding the meaning of these terms by all involved stakeholders including those who are not informed about the properties and mathematical representation of the associated models.

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[3] *Functionality* is the set of features (functions) pertaining to given complex object.