STABILITY GREEDY ALGORITHM FOR ONE PROBLEMS DISCRETE OPTIMIZATION

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In this paper we apply the theory of ordered convexity to stability of the greedy algorithm with respect to perturbations of the parameters of the coordinate-convex objective functions.

Let $Z^n = (Z^n, \leq)$ $(Z_+^n = (Z_+^n, \leq))$ be the set of all (nonnegative) integer $n-$vectors. If $0 = (0,...,0) \in P \subseteq Z_+^n$, P is finite, and the conditions $x \leq y$ and $x, y \in P$ imply the inclusion $[x, y] = \{z : x \leq z \leq y, z \in Z_+^n\} \subseteq P$ then the set P is called a finite ordered-convex set with zero [1]. In what follows, we assume that $P \subseteq Z_+^n$ is a finite ordered-convex set with zero.

A function $f : Z_+^n \to R$ (where R denotes the set of real numbers) is said to be ρ-coordinate-convex [2], if

$$\Delta_{ij} f(x) = \Delta_j f(x + e^i) - \Delta_j f(x) \leq 0, \forall x \in Z_+^n, i, j \in N = \{1,2,...,n\}, i \neq j,$$

$$\Delta_{ii} f(x) \leq -\rho_i, \forall x \in Z_+^n, i \in N,$$

where

$\Delta_j f(x) = f(x + e^j) - f(x), e^j = (e_1^j,...,e_n^j), e_j^j = 1, e_j^k = 0, j \neq k, j, k \in N,$

$\rho = (\rho_1,...,\rho_n) \in R_+^n,$

R_+^n - is the set of nonnegative real n-vectors.

We denote the set of all ρ-coordinate-convex functions by $\Re_\rho(Z_+^n)$.

A usual, a function $f : Z_+^n \to R$ is non- decreasing, if $\Delta_i f(x) \geq 0$ for any $x \in Z_+^n$ and $i \in N$.

Consider the discrete optimization Problem A:

$$\max\{f(x) : x = (x_1,...,x_n) \in P\},$$

where $f : Z_+^n \to R$ is a non-decreasing $\rho-$ coordinate-convex function, $P \subseteq Z_+^n$ - ordered-convexity set.

Let x^* be an optimal solution Problem A, and let x^g be its gradient solution, i.e., the point obtained by applying the gradient coordinate ascent algorithm (see. e.g. [1, 2]). By a guaranteed error estimate for the gradient algorithm in Problem A we mean a number $\varepsilon \geq 0$ for which

$$\frac{f(x^*) - f(x^g)}{f(x^*) - f(0)} \leq \varepsilon.$$

By perturbations of Problem A by means of a function $f(x)$ we mean the problems A^δ

$$\max\{f^\delta(x) : x = (x_1,...,x_n) \in P\},$$

where

$$f^{\delta}(x) \in \Re_q(Z_+^n), \left| c(f^{\delta}) - c(f) \right| \le \delta, \delta \in R_+^1,$$

$$c(f) = \min\left(\frac{\Delta_i f(x) - \Delta_j f(\pi_i(x))}{\Delta_i f(x)} : \Delta_i(f(x) > \Delta_j f(\pi_i(x)) \ge 0, i \in fes(x,P), j \in fes(\pi_i(x),P) \right\},$$

$$fes(x,P) = \{ i \in N : x + e^i \in P \}, \pi_i(x) = (x_1,...,x_{i-1}, x_i + 1, x_{i+1},...,x_n).$$

Let $\varepsilon\,(\varepsilon^{\delta})$ be a guaranteed error estimate for the gradient algorithm in some unperturbed (perturbed) discrete optimization problem. As usual, we say that the gradient algorithm is stable, if $\varepsilon^{\delta} < \varepsilon K(\delta)$, where $K(\delta) \to 1 \; as \; \delta \to 0$ [2].

Theorem. Let ε and ε^{δ} be guaranteed error estimates for the gradient algorithm in Problems A and A^{δ}, respectively, $c(f) < 1$. Then $\varepsilon^{\delta} < \varepsilon$.

References

1. M.M. Kovalev (1987) Matroids in discrete Optimization (in Russian), Minsk.
2. A.B. Ramazanov // Mathematical Notes, vol. 84, No. 1, pp. 147-151 (2008).