ALGORITHMS OF MOBILE CONTROL IN THE SYSTEMS WITH THE DISTRIBUTED PARAMETERS

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At present there is a sufficiently large number of works, dedicated to questions of mobile control of the thermal processes [1,2]. However, studies of mobile control in the oscillatory systems did not thus far obtain proper development and it is possible to indicate the very small number of works, dedicated to this direction [1,4,6]. Such studies can have both theoretical and applied value. Work [3] examines the tasks of optimal and instantly- optimum point control, in which the criterion of optimality is the energy, stored up in the system. By instantly-optimum is understood such control, with which at each moment is achieved the change in the criterion of optimality greatest in the absolute value, in this case - the energy, stored up in the system. In the work [4] are obtained the algorithms of mobile control for the distributed systems in the sliding regime with the use of Lyapunov function. Work [6] is dedicated to the development of the algorithms of instantly-optimum mobile control in the system, which consists of the massive particles, connected with springs.

Control algorithms for the sufficiently broad class of oscillatory systems with the distributed parameters in the present work are developed. It is shown that among the distributed controls limited on the resources instantly-optimum is mobile control, moreover the place of the application of action is the point, at which the impulse density is maximum in the absolute value. The results of the numerical computations of system with the developed control algorithms are given.

Formulation of the problem

Oscillating processes in the broad class of objects can be described by the following equation

$$\rho(x)\frac{\partial^2 Q}{\partial t^2} = div\big(a(x)gradQ\big) - qQ + F(t,x) = -LQ + F(t,x),\quad (x,t)\in G\times(0,T), \tag{1}$$

$$Q(0,x) = Q_0(x),\quad \left.\frac{\partial Q}{\partial t}\right|_{t=0} = Q_1(x),\quad x\in\bar{G}, \tag{2}$$

$$\alpha Q + \beta\frac{\partial Q}{\partial n} = 0,\quad t\ge 0,\quad x\in\Gamma. \tag{3}$$

Here $x\in R^n$ is the vector of space coordinates (usually $n = 1,2$ or 3), t is time, $Q(t,x)$ is the state of the object, which occupies region G, $\bar{G}$ is closing of G, Γ is boundary of the region G; $\rho(x) > 0$ is material density of object, $a(x) > 0$ - modulus of elasticity, $\alpha(x)\ge 0, \beta(x)\ge 0$ are the assigned functions, Γ_0 is that part of Γ, where $\alpha > 0$ and $\beta > 0$, $\partial Q/\partial n$ is directional derivative of external normal to $\bar{G}$, $F(t,x)$ function controlling action on the object is described.

By the equations of form (1) - (3) it is possible to describe the oscillations of string, membrane, elements of stationary constructions, transport, aviation and space constructions, acoustic phenomena, processes in the long lines and other real objects. We will examine the controlling actions of the following forms.

1. Distributed controlling action.

$$F(t,x) = p(t)w(t,x), \tag{4}$$

Where $p(t)$ is the intensity of action, $P_1 \le p(t) \le P_2$, $P_1 \le 0$ and $P_2 > 0$ are the given numbers, $w(t,x),\quad t\ge 0,\ x\in\bar{G}$, the function, which determines the distribution of action in the space. It is assumed that it is integrated by x with each t and it satisfies conditions $w(t,x) \geq 0$, $\int_G w(t,x)dx = 1$. We will include also δ - Dirac's function in the set of the permissible functions. controls are intensity the actions and the distribution of action in the space.

2. Mobile controlling action.

$$F(t,x) = p(t)\psi(x - s(t)) \tag{5}$$

where $p(t)$ is the intensity of action, $\psi(x)$ is the function of the form of mobile action, $\psi(x) \geq 0$, $\int_G \psi(x)dx = 1$, $\operatorname{supp}\psi(x) \subset G$, supp (supp - carrier of function). By examples to the functions there can be the function of Dirac, Gauss function $\psi(x) = (1/\sqrt{2\pi\sigma})\exp(-(1/2\sigma)x^2)$ with the sufficiently small values σ and other similar functions. The law of the motion of the center of action is described by the function $s(t)$, which is assumed to be that measured and are taken values in the region G.

3. Impulse mobile controlling action.

$$F(t,x) = \sum_{i=0}^{\infty} p_i(t)\psi(x - s_i), \quad p_i(t) = \begin{cases} p_i[1(t - iT) - 1(t - (i+\gamma)T)) & t \in [iT, (i+\gamma)T) \\ 0 & t \in [((i+\gamma)T, (i+1)T) \end{cases}, \tag{6}$$

Where $p_i(t)$ - impulse function for which they are determined: p_i - the value of intensity in impulse number i, T - the impulse repetition period, $\tau = \gamma T$, $0 < \gamma < 1$, the impulse duration; s_i - the position of the center of action in impulse number i. Function $\psi(x)$ and limitations to the intensity of action are determined with the description distributed and mobile controlling actions.

The following of problem of control is state.

Let $E(0) > 0$ - energy of system (1) - (3) at the of moment $t = 0$. Energy of system (1) - (3) at the moment t is defined as of the of sum of kinetic and potential of energies by of the following of formula [7].

$$E(t) = \frac{1}{2}\int_G \left[\rho\left(\frac{\partial Q}{\partial t}\right)^2 + a\left|gradQ\right|^2 + qQ^2 \right] dx + \frac{1}{2}\int_{\Gamma_0} a\frac{\alpha}{\beta}Q^2 ds. \tag{7}$$

It is necessary to find such a control $F(t,x)$, which depends on the state of the object, with which a change in the energy of system (1) - (3) takes the greatest (or the greatest in the absolute value negative) value. If a change in the energy $\partial E/\partial t$ has the greatest positive value, we obtain an increase in the energy of system. If the value $\partial E/\partial t < 0$ and greatest in the absolute value, we obtain the damping of system. We will call control, which satisfies the stated requirements, instantly-optimum (or local-optimum). For the task with impulse mobile control it is necessary to find such control, with which the value ΔE_i of an increase in the energy in action time each impulse $\tau = \gamma T$ greatest (or negative and greatest in the absolute value)

2. Algorithms of control

By the investigation of derivative of the function, which determines energy of the system (7) are obtained control algorithms with the feedback for the system (1) - (3) with the controls of form (4), (5) and (6). In particular, for the systems with distributed control of form (4) it is shown that mobile point control is local - optimum in this class of functions, and the place of the application of control is the point, at which the value $\rho(x)\dot{Q}(t,x)$ it reaches the greatest (or the greatest in the absolute value negative) value. In this case the intensity of instantly-optimum control takes maximum or minimum value. Control algorithm appears as follows:

$$w(t,x) = \delta(x - \hat{s}(t)), \quad \hat{s}(t) = \arg\max_{x \in G}\left|\rho(x)\dot{Q}(t,x)\right|, \quad \hat{p}(t) = \begin{cases} \pm P_2, & \rho(\hat{s}(t))\dot{Q}(t,\hat{s}(t)) > 0 \\ \mp P_1, & \rho(\hat{s}(t))\dot{Q}(t,\hat{s}(t)) < 0 \end{cases}. \tag{8}$$

Here $\hat{s}(t)$ - the instantly- optimum law of the motion of action, $\hat{p}(t)$ - the law of variation in the intensity. Plus sign corresponds to an increase of the energy in the system, minus sign - to vibration damping. Analogous algorithms are obtained for mobile control (5), impulse control (6), one-sided control ($P_1 = 0$, $P_2 > 0$).

Checking the developed algorithms was produced with the aid of the program, developed in the system of Matlab. The equation of oscillations with the zero boundary conditions and the following parameters one-dimensional on the space coordinates was simulated as the controlled object. The object had following parameters: $l = 5$, the length of object $l = 5$. Initial conditions were assigned in the form: $Q_0(x) = c_1 \sin((\pi / l)x) + c_2 \sin(2(\pi / l)x) + c_3 \sin(3(\pi / l)x) + c_4 \sin(4(\pi / l)x)$, $Q_1(x) = 0$, where $c_1,...,c_4$ - the given numbers. The solution of this equation was produced according to explicit difference circuit [5]. Was investigated system with the impulse mobile action of form (6). Was simulated the state of object during the different values of the managers of the parameters: the impulse repetition period $T = 0.05 \div 3$, the impulse duration $\tau = \gamma T = 0.005 \div 0.3$ ($\gamma = 0.9$). $\tau = \gamma T = 0.005 \div 0.3$ ($\gamma = 0.9$), $P_1 = -1, P_2 = 1$. Function of the form took the form:

$$\psi(x) = \begin{cases} (1/b)\cos^2(\pi/2b)x, & x \in (-b,b) \\ 0, & x \notin (-b,b) \end{cases}, \qquad (9)$$

where $b = 0.5$ - the half-width of the function of form. Were simulated both the algorithm of vibration damping and the algorithm of an increase in the energy, stored up in the system. In the report the results of the numerical computations are given: obtained the controlling actions $p(t)$ and $\hat{s}_i$, $i = 1,2,...,$ with the different values of the parameters and the corresponding states of the controlled object.

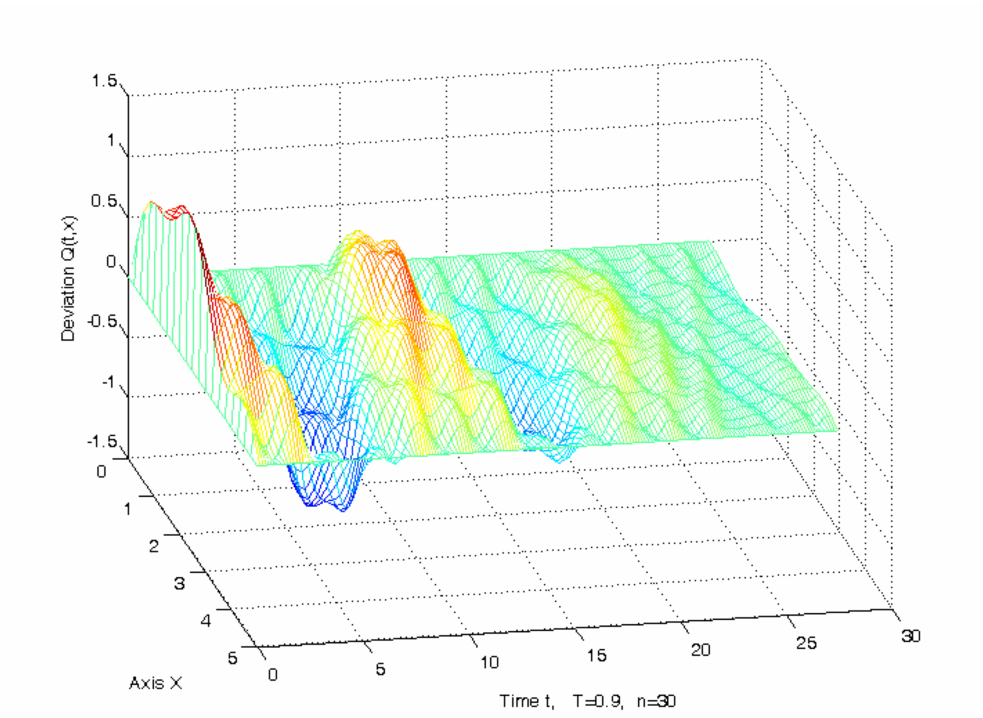

Fig.1

On fig.1 is shown an example of vibration damping with use of mobile action.

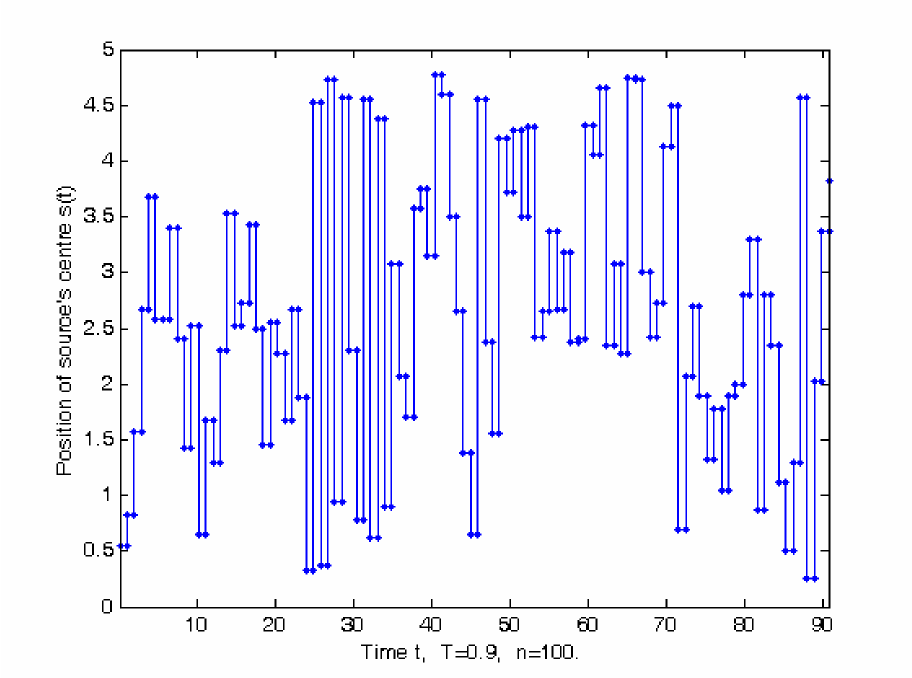

Fig.2

Fig.2 illustrates the law of the motion of the center of the action.

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