OPTIMAL CONTROL BY SOURCES IN DISTRIBUTED SYSTEMS ON THE CLASSES OF PIECE-WISE CONSTANT AND HEAVISIDE FUNCTIONS

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While controlling real-life objects, optimization of the control actions from the class of continuous and piecewise continuous functions causes some technical obstacles. The solution to optimal control problems on the classes of functions that are technically easily implemented is of important value [1-5]. The systems with control influences from the classes of piece-wise constant, in particularly Heaviside functions may be concerned to such classes.

In the mathematical models of many controlled processes, Heaviside step functions are used as control impacts. It is clear that this is a particular case of a piece-wise constant function, but the control using Heaviside functions is of interest from the practical point of view, since, in practice, many controlled processes are such that each impact takes on a value that is constant in time and is switched on only once.

The problems of optimal control are investigated on the classes of piece-wise constant and Heaviside functions in [3, 4], for the case of ordinary differential equations.

In this paper, the optimal control problems by concentrated sources in distributed systems, when the controls belong to such classes of functions as piece-wise constant and Heaviside are considered.

The interest to problems of motions of the sources in one or another meaning, with various physical natures has increased lately. For example, the moving sources of concentration of the chemical reagent, mass (pressure) of the underground waters, of the oil (in general matter), impulse, tension, thermo tension, heat, voice, radiation, electromagnet vibrations (in general of the energy), information and so on.

Adjoining with continuous displacement of the sources, there are some examples, when source may move from one position to another only by jumping, and it is necessary to find optimal motion control on the class of such jump illustrated displacements.

The optimal control problems by the motion of sources and their intensity (power) are investigated by many authors on the class of piece-wise continuous functions [1, 5]. The statement of the problem of control by concentrated sources for two dimensional case is investigated in the work [5], and in this circumstance optimization consists of defining as optimal law of motion of the sources (trajectories and the motion on it), as well as their intensity.

Let us consider the more general statement of optimal control problems by systems with distributed parameters, which consist of the minimization of the next functional

$$J(w) = \alpha_1 \int_{\Omega} [u(x,T;v) - U(x)]^2 dx + \alpha_2 \left\| v(t) - v^0(t) \right\|_{L_2}^2 \tag{1}$$

with conditions that, the position of controlling object is described by the n - dimensional boundary problem according to parabolic type:

$$u_t = div\big(\sigma(x) grad\, u(x,t)\big) + \sum_{i=1}^{L} v_i(t) b_i(x,t) \delta(x - \xi^i(t)), \quad x \in \Omega \subset R^n, 0 < t \le T, \tag{2}$$

$$u(x,0) = \varphi(x), \quad x \in \Omega, \tag{3}$$

$$u(x,t)|_{x\in\Gamma^1} = \mu_1(x,t), \sigma(x)\frac{\partial u(x,t)}{\partial N}|_{x\in\Gamma^2} = \mu_2(x,t), 0 < t \le T, \tag{4}$$

$$\Gamma = \Gamma^1 \cup \Gamma^2 = \partial\Omega, \Gamma^1 \cap \Gamma^2 = \varnothing.$$

Here $\dfrac{\partial u(x,t)}{\partial N} = u_{x_j} \cos nx_i$, n is a unique normal to Γ^2; $u = u(x,t) = u(x,t;v)$ is a phase position of the object, which is determining from the solution of the boundary problem of (2)–(4) on corresponding admissible value of optimized control vector $v(t) = (v_1(t),....,v_L(t))$; $v^0(t) = (v_1^0(t),....,v_L^0(t))$ -is a given vector-function; R^n – is n-dimensional Euclidian space; L – is a given number of control influences (sources); $\varphi(x), \mu_1(x,t), \mu_2(x,t), b_1(x,t),...,b_L(x,t)$, $\sigma(x)$, $U(x)$, $\alpha_i > 0, i = 1,2$, $l > 0, T > 0$ are given functions and values, determining the investigated process and the criterions of control on it; $\delta(x) = \prod_{i=1}^{n} \delta(x_i)$, $\delta(x_i)-$ is a generalized Dirac function.

It is interesting the case when the functions $\mu_1(x,t), \mu_2(x,t)$ named as boundary controls are optimized in the problem of (1)-(4).

According to position of sources $\xi^i(t) = (\xi_1^i(t),...,\xi_n^i(t)),\ i = 1,...,L$ may be considered the next variants:

1. The sources are motionless, i.e.,

$$\xi^i(t) = \xi^i = const,\ t \in [0,T],\ \xi^i \in R^n,\ i = 1,...,L.$$

2. The sources move and the trajectory of their motion in Ω are determined by piece-wise continuous functions

$$\xi^i(t), i = 1,...,L,\ t \in [0,T].$$

3. The systems of differential equations

$$\dot{\xi}^i(t) = f^i(\xi^i, s^i(t)),\ \xi^i(0) = \xi_0^i,\ i = 1,...,L, \tag{5}$$

determine the law of motion of the sources, where ξ_0^i, $i = 1,...,L$ are given initial values of trajectories of the sources; $f^i, i = 1,...,L$ – are given vector-functions; $s^i(t)$ –control influence on motion, at constrains $\underline{s} \le s^i(t) \le \overline{s}$, $i = 1,...,L$, i.e., the mechanical motion of sources also may be controlled processes.

The optimal control problems are investigated for various cases, according to position of the sources $\xi^i(t) = (\xi_1^i(t),...,\xi_n^i(t))$, in the work:

1. The coordinates of motionless sources - ξ^i, or the trajectories of motion of the sources - $\xi^i(t)$ are given; the process is controlled only by power of sources.

2. The process is controlled by power and by motion law of the sources: optimization of coordinates or trajectories of sources.

3. The process is controlled by the power of sources, defining by the system of differential equations (5).

The investigations according to these problems are carried out for the next two classes of control influences below.

1. Control influences are from the class of piece-wise constant functions:

$$v_i(t) = q_{ij} = const,\ \ t \in [\theta_{ij-1}, \theta_{ij}), \theta_{ij-1} < \theta_{ij},\ j = 1,...m_i,\ \theta_{i0} = 0,\ \theta_{im} = T,$$

$$M = \sum_{i=1}^{L} m_i, \qquad i = 1,...,L, \tag{6}$$

are determined by finite-dimensional vector $v = (q,\theta) \in R^{2LM}$, i.e., the values of controls $v_i(t)$ are constant on semi-intervals $[\theta_{ij-1}, \theta_{ij}) \subset [0,T]$ and must belong to some admissible set of V, in particularly, to the next parallelepiped

$$V = \{q_i = (q_{i1},...,q_{im_i}):\ \xi^i_j \le v^i_j \le \eta^i_j,\ \xi^i_j, \eta^i_j \in R,\ j = 1,...m_i,\ i = 1,...L\}, \quad (7)$$

and $\theta_{ij},\ j = 1,...,m_i - 1, i = 1,...,L$ are determine the intervals of constancy $[\theta_{ij-1}, \theta_{ij})$ of the power of i-th sources, $m_i -$ is a given number of impulsive influences of $i-$th source.

2. Control influences are from the class of functions Heaviside:

$$v_i(t) = q_i \chi(t - \theta_i),\ i = 1,...,L, \quad (8)$$

are assigned by finite-dimensional vector

$$v = (q,\theta) = (q_1,...,q_L,\theta_1,...,\theta_L) \in R^{2L}, \quad (9)$$

where $\chi(t-\theta_i)$ –is a Heaviside function, $i-$th component is a power of $i-$th source: $v_i = q_i$, beginning to influence at the moment of time $v_{L+i} = \theta_i$, $i = 1,..,L$.

Let us consider the following constraints on control parameters:

$$\underline{q_i} \le q_i \le \overline{q_i}, 0 \le \theta_i \le T, i = 1,...,L. \quad (10)$$

Here $\underline{q_i}, \overline{q_i}, L$ are given. So each component of control vector-function (control) $v(t)$ is piece-wise constant function with one change of value and is defined with values θ_i and q_i, i.e., with control's impact time and value: $v_i(t) = v_i(t; q_i, \theta_i)$, $i = 1,..,L$.

The classes of control functions as (6), (8) are used also in problems of boundary control according to $\mu_1(x,t), \mu_2(x,t)$, in the article.

It is considered that, the functions and parameters in the problem of (1)-(4) satisfy all conditions for existence and uniqueness of the boundary problem solution.

The considering optimal control problems are equivalent to the problems of the optimization of functional $J(v)$ in the closed admissible domain; so the set of optimal solutions is nonempty.

The controls may be discontinuous in the considering problems, so there isn't a classical solution of these problems. It is known from [6, 7] that on the even admissible control there is unique generalized solution of boundary problem (2)-(4).

It has been proved the theorem for prominence of functional in the considered classes of functions.

Theorem If the functional $J(v)$ is prominence in the class of piece-wise continuous functions then it is also prominence in the class of piece-wise constant functions.

The necessary conditions of optimality are obtained according to all considering problems of optimal control on the classes of piece-wise constant and Heaviside functions, which consist the analytical formulas for the gradient of functional (1) on the space of optimized parameters. These formulas allow to use the first order optimization methods to solve the problem of optimal control.

Analogical researches can be carried out as on the other statements of boundary problems, as well as on the processes described by other types of differential equations with private derivatives.

The results of the carried out numerical experiments and their analysis will be shown at lecture time.

References

1. A.G. Butkovskiy, L.M. Pustilnikov (1980) Theory of moving control by systems with distributed parameters (in Russian) M.: Nauka, p. 384.
2. G.A. Kolokolnikova, (1997). Variational Maximum Principle for Discontinuous Trajectories of Unbounded Asymptotically Linear Control Systems, Journal of Differential Equations, 33, 1633-1640.

3. K.R. Aida-Zade, E.R. Ashrafova (2009). Control of Systems with Concentrated Parameters in a Class of Special Control Functions, Automatic Control and Computer Science, 43(3), 150-157.
4. K.R. Aida-zade, A.B. Rahimov (2007). Solution of Optimal Control Problem in Class of Piece-wise constant Functions, Automatic Control and Computer Science, 41(1), 18-24.
5. K.R. Aida-zade, A.B. Handzel (1997). Two dimensional problems of control by concentrated sources in distributed systems (in Russian), Izvestiya ANAS, p. 81-85.
6. J.L. Lions (1987). Controle des systemes distributes singuliers. Gauthier, Willars.
7. O.A. Ladijenskaya (1973). The boundary problems of mathematical physics (in Russian), M. Nauka, p. 408.