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ON QUASI-SINGULAR CONTROLS IN AN OPTIMAL CONTROL PROBLEM DESCRIBED BY VOLTERRA'S DIFFERENCE EQUATION

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Consider a problem on minimum of the functional

$$S(u)=\varphi(z(t_1,x_1)) \tag{1}$$

under constraints

$$z(t+1,x+1)=\sum_{\tau=t_0}^{t}\sum_{s=x_0}^{x} f(t,x,\tau,s,z(\tau,s),u(\tau,s)), \tag{2}$$

$$(t,x)\in T\times X \quad (T=\{t_0,t_0+1,\dots,t_1-1\};\ X=\{x_0,x_0+1,\dots,x_1-1\}),$$

$$\begin{gathered} z(t_0,x)=a(x), \quad x\in X\cup x_1, \\ z(t,x_0)=b(t), \quad t\in T\cup t_1, \\ a(x_0)=b(t_0), \end{gathered} \tag{3}$$

$$u(t,x)\in U, \quad (t,x)\in T\times X\ . \tag{4}$$

Here $\varphi(z)$ is a twice continuously differentiable scalar function, $f(t,x,\tau,s,z,u)$ is a given n-dimensional vector-function continuous in the aggregate of all variables together with partial derivatives with respect to (z,u) to the second order inclusively, to t_0, t_1, x_0, x are the given numbers, the differences t_1-t_0, x_1-x_0 a natural numbers, $a(x)$, $b(t)$ are the given n-dimensional discrete vector-functions, U is a given non-empty, bounded and convex set, $u(t,x)$ is r-dimensional vector of control actions (admissible control).

The admissible control $(u(t,x),\ z(t,x))$ delivering minimum to the functional (1) under constraints (2)-(4) is called an optimal process, the control $u(t,x)$ an optimal control.

Assuming $(u(t,x),\ z(t,x))$ a fixed admissible process, we introduce the denotation

$$H(t,x,z(t,x),u(t,x))=\sum_{\tau=t}^{t_1-1}\sum_{s=x}^{x_1-1}\psi'(\tau,s)\, f(\tau,s,t,x,\tau,z(t,x),u(t,x)),$$

where $\psi=\psi(t,x)$ is n-dimensional vector-function being a solution of the boundary value problem

$$\psi(t-1,x-1)=H_z(t,x,z(t,x),u(t,x),\psi(t,x)), \tag{5}$$

$$\begin{gathered} \psi(t_1-1,x-1)=0\,, \quad x=x_0,x_0+1,\dots,x_1, \\ \psi(t-1,x_1-1)=0\,, \quad x=t_0,t_0+1,\dots,t_1, \\ \psi(t_1-1,x_1-1)=-\varphi_z(z(t_1,x_1)). \end{gathered} \tag{6}$$

The boundary value problem (5)-(6) is called conjugated to the problem (1)-(4).

In the paper, at first it was proved that if the set U is convex, for optimality of the admissible control $u(t,x)$ in problem (1)-(4) the inequality

$$\sum_{t=t_0}^{t_1-1}\sum_{x=x_0}^{x_1-1} H_u'(t,x,z(t,x),u(t,x),\psi(t,x))(v(t,x)-u(t,x))\le 0\,, \tag{7}$$

should be fulfilled for all $v(t,x)\in U$, $(t,x)\in T\times X$.

Inequality (7) is a first order necessary optimality condition in the form of linearized maximum principle [1-3].

Further, the case of degeneration of necessary optimality condition (7) is studied.

Definition. The admissible control $u(t,x)$ is said to be quasi-singular if for all $v(t,x)\in U$, $(t,x)\in T\times X$

$$\sum_{t=t_0}^{t_1-1}\sum_{x=x_0}^{x_1-1} H_u'(t,x,z(t,x),u(t,x),\psi(t,x))(v(t,x)-u(t,x))\equiv 0 .$$

Let $y(t,x)$ be a solution of the boundary value problem

$$y(t+1,x+1)=$$
$$=\sum_{\tau=t_0}^{t}\sum_{s=x_0}^{x}\left[f_z(t,x,\tau,s,z(\tau,s),u(\tau,s))y(\tau,s)+f_u(t,x,\tau,s,z(\tau,s),u(\tau,s))(v(\tau,s)-u(\tau,s))\right], \tag{8}$$

$$\begin{aligned} y(t_0,x)&=0, \quad x\in X\cup x_1 ,\\ y(t,x_0)&=0, \quad t\in T\cup t_1 . \end{aligned} \tag{9}$$

We call problem (8)-(9) an equation in variations in problem (1)-(4).

Developing the scheme suggested in [4-6], we get various necessary optimality conditions of singular controls. Cite one of them:

Theorem. If the set U is convex, for optimality of the quasi-singular control $u(t,x)$ in problem (1)-(4) the inequality

$$\begin{aligned} y'(t_1,x_1)\varphi_{zz}(z(t_1,x_1))y(t_1,x_1)-\sum_{t=t_0}^{t_1-1}\sum_{x=x_0}^{x_1-1}\Big[y'(t,x)H_{zz}(t,x,z(t,x),u(t,x),\psi(t,x))y(t,x)+\\ +2(v(t,x)-u(t,x))'H_{uz}(t,x,z(t,x),u(t,x),\psi(t,x))y(t,x)+\\ +(v(t,x)-u(t,x))'H_{uu}(t,x,z(t,x),u(t,x),\psi(t,x))(v(t,x)-u(t,x))\Big]\ge 0 , \end{aligned} \tag{10}$$

should be fulfilled for all $v(t,x)\in U$, $(t,x)\in T\times X$.

Inequality (10) is an implicit necessary optimality condition of quasisingular controls and is of sufficiently general character.

Using inequality (10), by means of the scheme developed in [4-6], we could get explicit necessary optimality conditions that are directly expressed by the parameters of problem (1)-(4). Different special cases are studied.

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