INVESTIGATION OF QUASI-SINGULAR CONTROL IN THE MOISEYEV PROBLEM

Ilaha Nagiyeva

Cybernetics Institute of ANAS, Baku, Azerbaijan
ilaha_21@mail.ru

It is required to find the minimum of the functional

$$S(u)=\varphi(x(t_1))+\int_{t_0}^{t_1}\int_{t_0}^{t_1}G(t,s,x(t),x(s),u(t),u(s))ds\,dt\,, \tag{1}$$

under constraints

$$u(t)\in U\subset R^r,\quad t\in T=[t_0,t_1], \tag{2}$$

$$\dot{x}=f(t,x,u),\quad t\in T,\quad x(t_0)=x_0. \tag{3}$$

Here, $\varphi(x)$ is a twice continuously differentiable scalar function, $G(t,s,a,b,u,v)$ is given scalar function continuous in the aggregate of variables together with partial derivatives with respect to (a,b,u,v) to the second order inclusively, $f(t,x,u)$ is a given n-dimensional vector-function continuous in the aggregate of variables together with partial derivatives with respect to (x,u) to the second order inclusively, t_0, t_1, x_0 are given, U is a given non-empty, bounded and convex set, $u=u(t)$ is r-dimensional piecewise-continuous (with finite number first order continuity points) control vector-function (admissible control).

Assuming $(u(t),\ x(t))$ a fixed admissible process, we introduce the denotation

$$\begin{aligned}
H(t,x,u,\psi)&=\psi' f(t,x,u),\\
H_x[t]&\equiv H_x(t,x(t),u(t),\psi(t)),\\
H_u[t]&\equiv H_u(t,x(t),u(t),\psi(t)),\\
H_{uu}[t]&\equiv H_{uu}(t,x(t),u(t),\psi(t)),\\
G_x[t,s]&\equiv G_x(t,s,\ x(t),x(s),u(t),u(s)),\\
G_u[t,s]&\equiv G_u(t,s,\ x(t),x(s),u(t),u(s)),
\end{aligned}$$

where $\psi=\psi(t)$ is a solution of the conjugated system

$$\dot{\psi}=-H_x(t,x(t),u(t),\psi)+\int_{t_0}^{t_1}[G_a[t,s]+G_b[s,t]]ds\,,$$

$$\psi(t_1)=-\varphi_x(x(t_1)).$$

In the paper, a formula for the increment of the quality test is a constructed and various linearized and quadratic first and second order optimality conditions are obtained.

Cite some of the them.

Theorem 1. For optimality of the admissible control $u(t)$ in problem (1)-(3), the inequality

$$\left(H_u[\theta]-\int_{t_0}^{t_1}[G_u[\theta,s]+G_v[s,\theta]]ds\right)'(v-u(\theta))\le 0 \tag{4}$$

should be fulfilled for all $v\in U$ and $\theta\in[t_0,t_1)$.

Relation (4) is a first order linearized necessary optimality condition and is an analogy of the linearized maximum condition [1, 2].

Definition. We call the admissible control $u(t)$ a quasi-singular control if for all $\theta \in [t_0, t_1)$ and $u \in U$

$$\left(H_u[\theta] - \int_{t_0}^{t_1} [G_u[\theta, s] + G_v[s, \theta]] ds \right)' (v - u(\theta)) = 0 .$$

Theorem 2. For optimality of the quasi-singular control $u(t)$, the inequality

$$(v - u(\theta))' \left(H_{uu}[\theta] - \int_{t_0}^{t_1} [G_{uu}[\theta, s] + G_{vv}[s, \theta]] ds \right)' (v - u(\theta)) \le 0$$

should be fulfilled for all $v \in U$ and $\theta \in [t_0, t_1)$.

Further, we get more general necessary optimality conditions of quasi-singular controls that is the generalization of appropriate results from [3] for the case of problem (1)-(3).

In deriving necessary optimality conditions we used the method suggested in [4, 5].

References

1. N.N. Moiseyev. Elements of theory of optimal systems. M. Nauka, 1975, 574 p.
2. R. Gabasov, F.M. Kirillova. Maximum principle in optimal control problem. Minsk. Nauka I technika. 1974, 272 p.
3. I.F. Nagiyeva. Quasi-singular controls in Moiseyev's control problem // Vestnik BSU, ser. fiz. mat. nauk, 2006, № 4, pp. 34-38.
4. K.B. Mansimov. Singular controls in delay systems. Baku, ELM. 1999, 176 p.
5. K.B. Mansimov. Necessary optimality conditions of singular processes in optimal control problems // Author's thesis for Doctor's degree. Baku, 1994, 34 p.