RELATION BETWEEN THE SETS OF SOLUTIONS OF INITIAL AND SALIENT PROBLEMS IN PROCESSES GOVERNED BY PARABOLIC TYPE EQUATIONS

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Let us consider a problem of minimum of the following functional

$$J_0(z,u)=\int_0^T\int_0^X f_0(t,x,z(t,x),z_x(t,x),u(t,x))dtdx\,,$$

determined on the solutions of the following problem:

$$\tfrac{\partial z}{\partial t}-\sum_{i,j=1}^{n}\tfrac{\partial}{\partial x_i}(a_{ij}(t,x)\tfrac{\partial z}{\partial x_j}+a_i(t,x)z)+\sum_{i=1}^{n}b_i(t,x)z_{x_i}+a(t,x,u)z=f(t,x,u), \qquad (1)$$

$$\begin{aligned} &z(0,x)=\beta(x), x\in[0,X],\\ &z\big|_{\partial D}=0, t\in[0,T], \end{aligned} \qquad (2)$$

where $u(t,x)=(u_1(t,x),u_2(t,x),...,u_r(t,x))$ is a control vector-function. As a class "ordinary" admissible control we take measurable and bounded $u(t,x)$ accepting the values from $U\subset R^r$ .Let us denote a class of such controls by σ_U .

It is supposed that the functions $a_{ij}(t,x),a_i(t,x)$ are measurable functions given in D and the following conditions are satisfied:

$$\nu\xi^2\le a_{i,j}(t,x)\xi_i\xi_j\le\mu\xi^2,\nu,\mu=const>0.$$

But the functions $b_i(t,x)$ are continuous on D, the functions $a(t,x,u), f(t,x,u)$ are continuous on $D\times U$.

Moreover, the conditions from [1], that provide existence of solution of problem $(1),(2)$ in $W_2^{0,1}(D)$ are satisfied for each "ordinary" admissible control $u=u(t,x)$.

Let us denote the set of these solutions by G_0.

As a generalized control we take a finite weakly measurable [2] family of Radon measures μ_{tx} concentrated on U. And consider a problem of minimum of the following functional:

$$\mathrm{I}_0(z,\mu)=\int_0^T\int_0^X < f_0(t,x,z(t,x),z_x(t,x),u(t,x)),\mu_{tx}> dtdx\,,$$

determined on the solutions of the following equation: .

$$\tfrac{\partial z}{\partial t}-\tfrac{\partial}{\partial x_i}(a_{ij}\tfrac{\partial z}{\partial x_j}a_{iz})+b_i(t,x)\tfrac{\partial z}{\partial x_i}+a(t,x,u)z=< f(t,x,u),\mu_{tx}>\,, \qquad (3)$$

where

$$\langle \cdot, \mu_{tx} \rangle = \int_{R^r} (\cdot) d\mu_{tx} .$$

The problem $(3),(2)$ is called a salient problem. Let us denote the set of solutions of the problem $(3),(2)$ by G and let us denote the class of generalized controls by Ω_U.

In this work, we prove the following theorems establishing relation between the sets G_0 and G.

Every element of the set $G \setminus G_0$ is called a generalized condition, but the pair $(z, \mu_{tx}) \in (G \setminus G_0) \times (\Omega_U \setminus \sigma_U)$ is called slipping condition, here z corresponds to μ_{tx}.

Theorem 1: When the conditions mentioned abiul are satisfied, the set G is weakly closed in $W_2^{0,1}(D)$.

Theorem 2: Weak closure of the set G_0 coincides with the set G in $W_2^{0,1}(D)$, i.e. $\overline{G_0} = G$.

References

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2. Qamkrelidze R.V. The bases of optimal control. –Tbilisi, 1975.