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ON ONE OPTIMAL CONTROL PROBLEM

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We consider a problem on minimum of the functional

$$S(u) = \varphi(z(t_1, x_1)) \tag{1}$$

under constraints

$$z(t+1, x+1) = \sum_{\tau=t_0}^{t} \sum_{s=x_0}^{x} f(t, x, \tau, s, z(\tau, s), u(\tau, s)), \tag{2}$$

$$(t, x) \in T \times X \quad (T = \{t_0, t_0 + 1, ..., t_1 - 1\}; \ X = \{x_0, x_0 + 1, ..., x_1 - 1\}),$$

$$z(t_0, x) = a(x), \quad x \in X \cup x_1,$$

$$z(t, x_0) = b(t), \quad t \in T \cup t_1, \tag{3}$$

$$a(x_0) = b(t_0),$$

$$u(t, x) \in U, \quad (t, x) \in T \times X. \tag{4}$$

Here, $\varphi(z)$ is a given twice continuously differentiable scalar function, $f(t, x, \tau, s, z, u)$ is a given n-dimensional vector-function continuous in the aggregate of variables together with partial derivatives with respect to z to the second order inclusively, t_0, t_1, x_0, x are the given numbers, moreover the differences $t_1 - t_0$, $x_1 - x_0$ are natural numbers, $a(x)$, $b(t)$ are the given n-dimensional discrete vector-functions, U is a given non-empty and bounded set, $u(t, x)$ is r-dimensional vector of control actions (admissible control).

The admissible process $(u(t, x), z(t, x))$ delivering minimum to the functional (1) under constraints (2)-(4) is called an optimal process, the control $u(t, x)$ an optimal control.

Assuming $(u(t, x), z(t, x))$ a fixed admissible process, introduce the denotation

$$H(t, x, z(t, x), u(t, x)) = \sum_{\tau=t}^{t_1-1} \sum_{s=x}^{x_1-1} \psi'(\tau, s) f(\tau, s, t, x, \tau, z(t, x), u(t, x)),$$

$$\Delta_{v(t,x)} H(t, x, z(t, x), u(t, x), \psi(t, x)) \equiv H(t, x, z(t, x), v(t, x), \psi(t, x)) -$$

$$- H(t, x, z(t, x), u(t, x), \psi(t, x)),$$

where $\psi_i = \psi_i(t, x)$ n-dimensional vector-function being a solution of the problem

$$\psi(t-1, x-1) = H_z(t, x, z(t, x), u(t, x), \psi(t, x)), \tag{5}$$

$$\psi(t_1 - 1, x - 1) = 0,$$

$$\psi(t - 1, x_1 - 1) = 0, \tag{6}$$

$$\psi(t_1 - 1, x_1 - 1) = -\varphi_z(z(t_1, x_1)).$$

Boundary value problem (5)-(6) is called conjugated to problem (1)-(4).

It is proved in the paper that of the set

$$f(t, x, \tau, s, z(\tau, s), U) = \{\alpha : \alpha = f(t, x, \tau, s, z(\tau, s), v), \ v \in U\} \tag{7}$$

is convex, then for optimality of the admissible control $u(t, x)$ in problem (1)-(4), the inequality

$$\sum_{t=t_0}^{t_1-1} \sum_{x=x_0}^{x_1-1} \Delta_{v(t,x)} H(t, x, z(t, x), u(t, x), \psi(t, x)) \leq 0 \tag{8}$$

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should be fulfilled for all $v(t,x) \in U$, $(t,x) \in T \times X$.

Inequality (8) is a first order necessary optimality condition in the form of the discrete maximum principle [1-4].

Study the degeneration case of optimality condition (8).

Following f.e. [5, 6], introduce

Definition. The admissible control $u(t,x)$ is said to be singular in the sense of Pontryagin's maximum principle if for all $v(t,x) \in U$, $(t,x) \in T \times X$

$$\sum_{t=t_0}^{t_1-1} \sum_{x=x_0}^{x_1-1} \Delta_{v(t,x)} H(t,x,z(t,x),u(t,x),\psi(t,x)) \equiv 0.$$

Let $y(t,x)$ be a solution of the boundary value problem

$$y(t+1, x+1) =$$

$$= \sum_{\tau=t_0}^{t} \sum_{s=x_0}^{x} \left[f_z(\tau,s,t,x,z(\tau,s),u(\tau,s)) y(\tau,s) + \Delta_{v(\tau,s)} f(t,x,\tau,s,z(\tau,s),u(\tau,s)) \right], \tag{9}$$

$$y(t_0,x) = 0, \quad x \in X \cup x_1,$$
$$y(t,x_0) = 0, \quad t \in T \cup t_1. \tag{10}$$

We call boundary value problem (9)-(10) an equation in variations in problem (1)-(4).

Developing the scheme suggested in the papers [1-3], various necessary optimality conditions of singular controls are obtained. Cite one of them.

Theorem. If the set (7) is convex, then for optimality of singular, in the sense of Pontryagin's maximum principle, control $u(t,x)$ in problem (1)-(4) the inequality

$$y'(t_1,x_1) \varphi_{zz}(z(t_1,x_1)) y(t_1,x_1) - \sum_{t=t_0}^{t_1-1} \sum_{x=x_0}^{x_1-1} \left[y'(t,x) H_{zz}(t,x,z(t,x),u(t,x),\psi(t,x)) y(t,x) + \right.$$

$$\left. + 2\Delta_{v(t,x)} H'_z(t,x,z(t,x),u(t,x),\psi(t,x)) y(t,x) \right] \geq 0, \tag{11}$$

should be fulfilled for all $v(t,x) \in U$, $(t,x) \in T \times X$.

Inequality (11) is sufficiently general, but at the same time implicit necessary optimality condition of singular, in the sense of Pontryagin's maximum principle, controls.

Based on it, under some additional assumptions we succeeded to get necessary optimality conditions of singular controls obviously expressed by the parameters of problem (1)-(4).

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