NECESSARY OPTIMALITY CONDITIONS IN ONE DISCRETELY CONTINUOUS CONTROL PROBLEM

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Let the controlled system be described by the following system of equations

$$\frac{dx(k,t)}{dt} = f\left(k,t,x(k,t),x(k-1,t),u(k,t)\right),\quad (k,t)\in D=\{1\le k\le N,\ t_0\le t\le t_1\} \quad (1)$$

with the boundary conditions

$$\begin{gathered} x(k,t_0)=h(k),\quad k=\overline{0,N}, \\ x(0,t)=g(t),\quad t\in[t_0,t_1], \\ h(0)=g(t_0). \end{gathered} \quad (2)$$

Here t_0, t_1, N (a natural numbers) are given, $h(k)$, $g(t)$ are the given vector-functions, $f(k,t,x,a,u)$ is a given n-dimensional vector-function continuous in the aggregate of all variables together with partial derivatives with respect to x, $u(k,t)$ is a given discrete in k and piecewise-continuous in t, r-dimensional vector-function with values from the given non-empty and bounded set U , i.e.

$$u(k,t)\in U,\quad (k,t)\in\{1\le k\le N,\ t_0\le t\le t_1\}. \quad (3)$$

Such vector-functions are said to be admissible controls. The problem requires to minimize the terminal functional

$$S(u)=\sum_{k=1}^{N}\varphi(x(k,t_1)), \quad (4)$$

determined on the solutions of boundary value problem (1)-(2) generated by all possible admissible controls.

Here, $\varphi(x)$ is a given continuously-differentiable scalar function.

The admissible control $u(k,t)$ delivering minimum to the function (4) under restraints (1)-(3) is said to be an optimal control, the appropriate process $(u(k,t),\ x(k,t))$ an optimal process.

Assuming $(u(k,t),\ x(k,t))$ a fixed admissible control, introduce the denotation

$$H\left(k,t,x(k,t),x(k-1,t),u(k,t),\psi(k,t)\right)=\psi'(k,t)\,f\left(k,t,x(k,t),x(k-1,t),u(k,t)\right),$$

$$\begin{aligned} \Delta_{v(k,t)}H\left(k,t,x(k,t),x(k-1,t),u(k,t),\psi(k,t)\right) &= H\left(k,t,x(k,t),x(k-1,t),v(k,t),\psi(k,t)\right)- \\ &- H\left(k,t,x(k,t),x(k-1,t),u(k,t),\psi(k,t)\right). \end{aligned}$$

Here, $\psi(k,t)$ is n - dimensional vector-function of conjugated variables being a solution of the problem

$$\psi(k,t_1)=-\frac{\partial\varphi(x(k,t_1))}{\partial x},\quad k=\overline{1,N},$$

$$\dot\psi(k,t)=-\frac{\partial H\left(k,t,x(k,t),x(k-1,t),u(k,t),\psi(k,t)\right)}{\partial x}-$$

$$-\frac{\partial H\left(k+1,t,x(k+1,t),x(k,t),u(k+1,t),\psi(k+1,t)\right)}{\partial a},\quad k=\overline{1,N-1},$$

$$\dot{\psi}(N,t) = -\frac{\partial H(N,t,x(N,t),x(N-1,t),u(N,t),\psi(N,t))}{\partial x}.$$

Theorem. For optimality of the admissible control $u(k,t)$ in the considered problem, the inequality

$$\sum_{k=1}^{N}\left[H(k,\theta,x(k,\theta),v(k),\psi(k,\theta)) - H(k,\theta,x(k,\theta),u(k,\theta),\psi(k,\theta))\right] \le 0$$

should be fulfilled for all $v(k) \in U$, $k = \overline{1,N}$.

The theorem is the analogy of Pontryagin's maximum principle for the considered problem.

References

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