INVESTIGATION OF ONE DISCRETE CONTROL PROBLEM WITH INEQUALITY TYPE FUNCTIONAL CONSTRAINTS

Rasmiya Amirova[1], Kamil Mansimov[2]

[1]Cybernetics Institute of ANAS, Baku Azerbaijan
[2]Baku State University, Baku, Azerbaijan
[1]*amir.208@mail.ru*, [2]*mansimov@front.ru*

Consider a problem on minimum of the functional

$$S(u)=\varphi(z(t_1,x_1)) \tag{1}$$

under constraints

$$u(t,x)\in U, \quad (t,x)\in T\times X\ . \tag{2}$$

$$S_i(u)=\varphi_i(z(t_1,x_1)), \quad i=\overline{1,p}, \tag{3}$$

$$z(t+1,x+1)=\sum_{\tau=t_0}^{t}\sum_{s=x_0}^{x} f\,(t,x,\tau,s,z(\tau,s),u(\tau,s)),$$

$$(t,x)\in T\times X \quad (T=\{t_0,t_0+1,\ldots,t_1-1\};\ X=\{x_0,x_0+1,\ldots,x_1-1\}),$$

$$z(t_0,x)=a(x), \qquad x\in X\cup x_1,$$

$$z(t,x_0)=b(t), \qquad t\in T\cup t_1, \tag{4}$$

$$a(x_0)=b(t_0),$$

Here, $\varphi_i(z)$, $i=\overline{0,p}$ are the given twice continuously differentiable scalar functions, $f\,(t,x,\tau,s,z,u)$ is the given n-dimensional vector-function continuous in the aggregate of variables together with partial derivatives with respect to z to the second order inclusively, t_0, t_1, x_0, x are the given numbers, moreover the differences t_1-t_0, x_1-x_0 are natural numbers, $a(x)$, $b(t)$ are the given n-dimensional discrete vector-functions, U is the given non-empty and bounded set, $u(t,x)$ is a r - dimensional vector of control actions (admissible control).

If the solution $z(t,x)$ of boundary value problem (4) that corresponds to the admissible control $u(t,x)$ satisfies the constraints (3) such a control is said to be admissible, and the corresponding process an admissible process.

The process $(u(t,x),\ z(t,x))$ delivering minimum to the functional (1) under constraints (2)-(4) is said to be an optimal process, the control $u(t,x)$ an optimal control.

Considering $(u(t,x),\ z(t,x))$ as a fixed admissible process, introduce the denotation

$$H(t,x,z(t,x),u(t,x),\psi_i(t,x))=\sum_{\tau=t}^{t_1-1}\sum_{s=x}^{x_1-1}\psi_i'(\tau,s)\,f\,(\tau,s,t,x,\tau,z(t,x),u(t,x)),$$

$$\Delta_{v(t,x)}H(t,x,z(t,x),u(t,x),\psi_i(t,x))\equiv H(t,x,z(t,x),v(t,x),\psi_i(t,x))-$$

$$-H(t,x,z(t,x),u(t,x),\psi_i(t,x)), \quad i=\overline{0,p},$$

$$I(u)=\{i:\varphi_i(z(t_1,x_1))=0,\ i=\overline{1,p}\}, \quad J(u)=\{0\}\cup I(u),$$

where $\psi_i = \psi_i(t,x)$ is an n - dimensional vector-function being a solution of the problem

$$\psi_i(t-1,x-1) = H_z(t,x,z(t,x),u(t,x),\psi_i(t,x)), \tag{5}$$

$$\begin{gathered} \psi_i(t_1-1,x-1)=0, \\ \psi_i(t-1,x_1-1)=0, \\ \psi_i(t_1-1,x_1-1) = -\frac{\partial \varphi_i(z(t_1,x_1))}{\partial z}. \end{gathered} \tag{6}$$

Boundary value problem (5)-(6) is said to be conjugated to problem (1)-(4).

In the paper, by means of the results from [1-4], it is proved that if the set

$$f(t,x,\tau,s,z(\tau,s),U) = \{\alpha : \alpha = f(t,x,\tau,s,z(\tau,s),v),\ \ v\in U\} \tag{7}$$

is convex, then for the optimality of the admissible control $u(t,x)$ in problem (1)-(4), the inequality

$$\min_{i\in J(u)} \sum_{t=t_0}^{t_1-1} \sum_{x=x_0}^{x_1-1} \Delta_{v(t,x)} H(t,x,z(t,x),u(t,x),\psi_i(t,x)) \le 0, \tag{8}$$

should be fulfilled for all $v(t,x)\in U$, $(t,x)\in T\times X$.

Inequality (8) is a first order necessary optimality condition in the form of the discrete maximum principle.

Further, the case of degeneration of optimality condition (8) is studied.

References

1. K.B. Mansimov. Discrete systems. Baku, BSU, 2002, 114 c. (Russian)
2. K.B. Mansimov. Optimization of a class of discrete two-parametric systems // Differen. Uravnenia, 1991, № 2, pp. 359-362. (Russian)
3. K.B. Mansimov. Necessary optimality conditions second order in discrete two-parametric systems // Izv. AN Azerb. Ser. fiz.-tekhn. i mat. nauk, 1998, № 2, pp. 56-60. (Russian)
4. S.Ya. Gorokhovik. Necessary conditions of optimality of special controls in discrete systems with terminal constraints // Izv. AN BSSR, Ser. fiz.-tekhn. i mat. nauk, 1985, № 3, pp. 48-51. (Russian)