OPTIMAL CONTROL FOR HYBRID SYSTEM

Shahlar Maharramov[1], Zeynep Ors[2], Asli Guler[3]

[1-3]Yasar University, Bornova, Izmir, Turkey
[1]Cybernetics Institute of ANAS, Baku, Azerbaijan
[1]*shahlar.maharramov@yasar.edu.tr*, [2]*zeynep.ors@yasar.edu.tr*, [3]*asli.guler@yasar.edu.tr*

Abstract. In this short paper formulates an approach for solving optimal control problems for switching systems. In general, in such problems one needs to find both optimal continuous inputs and optimal switching sequence. In the paper it has shown difference between the article and without ant initial restriction and switching conditions, build methodology to find switching points and control. The main aim of this article compare the method which is published in the article [1] by K.Zakharov and the method which will be give below by author.

Problem formulation.
To understand presented problem deeply, author prefer for the interesting readers the articles [1], [4],[6], [7], [10] and [11].
We consider the following optimization problem:

$$\dot{x}_k(t) = f_k(t, x_k, u_k),\ \mathrm{u_k} \in U_k,\ x_1(t_0) = x_0 \tag{1}$$

$$\text{for} \quad t \in \Delta_k = [t_{k-1}, t_k],\ k = 1,2,3,...,N$$

here $t_1, t_2, ..., t_{N-1}, t_N$ are unknowns real numbers.
We will to tray to find minimum of following functional:

$$S(u_1, u_2, ...u_N, t_1, t_2, ..., t_N) = \sum_{k=1}^{N} \varphi_k(x_k(t_k)) \tag{2}$$

In this problem, $f_k : R \times R^n \times R^r \to R^n$, is continuous, at least continuously partially differentiable vector-valued functions with respect to its coordinates, $\varphi_k(x_k(t))$ is given continuously differentiable functions, u_k(t): R → $\mathrm{U_k} \subset \mathrm{R^r}$ are controls. The sets U_k, are assumed to be nonempty and bounded. The problem is that on the time interval $[t_0, t_N]$ consider the optimal control problem (1)-(2), to find controls $u_1, u_2, ..., u_N$, switching points $t_0, t_1, t_2, ..., t_{N-1}, t_N$, with corresponding state $x_{1,} x_2, ..., x_N$ satisfying (1) and (2) takes minimum value. Le us denote by $u = (u_1, u_2, ..., u_N)$, $x = (x_1, x_2, ..., x_N)$ and $t = (t_1, t_2, ..., t_N)$. Let us denote collection all all t and u, correcpondely by T and U.

Such kind of switching optimal control problem were used by the author [11] but the in the article [10] author didn't use initial conditions, reduce all no fixed switching instants to the fixed interval.

Let us give theorem to help find switching instants and control after getting necessary conditions for the above mentioned problem.

Theorem. If an optimal solution (t^*, u^*) exist for the problem (1)-(2) and for any given switching sequence t, there exist a corresponding $\hat{u}^* = u_t^*$ such that $S(\hat{t}, \hat{u})$ minimized then the following relation holds

$$\min_{t \in T, u \in U} S(t, u) = \min_{t \in T} \min_{u \in U} S(t, u) \tag{3}$$

Proof. First, we can assume that

$$\min_{t\in T,u\in T} S(t,u) \le \inf_{t\in T} \min_{u\in U} S(t,u) \tag{4}$$

It is obviously, because for any fixed t, there exist u_t^* such that $S(t,u_t^*) = \min_{u\in U} S(t,u)$. In same time for every pair (t,u_t^*), we can get $S(t^*,u^*) \le S(t,u_t^*)$, therefore from (4) we have

$$S(t^*,u^*) \le \inf_{t\in T} S(t,u_t^*) = \inf_{t\in T} \min_{u\in U} S(t,u) \tag{5}$$

We have also following inequality (to prove correctness this inequality is simple)

$$\inf_{t\in T} \min_{u\in U} S(t,u) \le \min_{u\in U} S(t^*,u) = S(t^*,u_{t^*}^*) \tag{6}$$

In (6) we can choose $u_{t^*}^* = u^*$, since for any other u, we must have $S(t^*,u^*) \le S(t^*,u)$ due to optimality of (t^*,u^*). Hence combining (5) and (6) we have

$$S(t^*,u^*) \le \inf_{t\in T} \min_{u\in U} S(t\ \ ,u) \le S(t^*,u_{t^*}^*) = S(t^*,u^*) \tag{7}$$

Hence all inequalities in (7) and the $\inf\limits_{t}$ can be replaced by $\min\limits_{t}$ so we obtain

$$S(t^*,u^*) = \min_{t\in T,u\in U} S(t\ \ ,u) = \min_{t\in T} \min_{u\in U} S(t,u) \tag{8}$$

Q.E.D

Remark1. *The right-hand of (8) needs twice the minimization process. It means that the above mentioned optimization problem reduces to simple form as a described in the right-hand side of the relation (8). This support the validity of the following two step optimization methodology. The necessary conditions for the problem(1) and (2) can be find in the articles in ref.[5] or [10].*

Remark2. *In this problem we didn't fixed switching instants. Such kind of problem is also investigated in the article [10] by the Dmitruk A and author gained optimality conditions and shoved that this hybrid optimal control in no fixed switching instants can be reduce to the to the fixed time interval , after this it can be applied Pontryagin maximum principle, see ref. [10].*

Let us describe following steps for the minimize cost functional after getting necessary conditions for the problem (1) and (2).

Step I. Let us at fist step fix t, and solve optimization problem

Step II. In second step, we can consider cost function as a function of the variable t, i.e., as a function

$$S_1 = S_1(t) = \min_{u\in U} S(t,u)$$

And then minimize S_1 with respect to t.

Such algorithms is used also in the article [11].

Illustrative example.

Consider the following two step time optimal control problem

$$\dot{x}_1 = x_2,\ \ \dot{x}_2 = u, \quad 0 \le t \le \tau\ ;$$

$$\dot{y}_1 = y_2,\ \dot{y}_2 = -y_2,\ \dot{y}_3 = y_4,\ \dot{y}_4 = v - y_4,\ \tau \le t \le T$$

The problem is to find switching point τ and endpoint T which to arrive the shortest time from point $(x_1(0), x_2(0)) = (c;0)$ to the point $(y_1(T), y_2(T)) = (0;0)$, where $c > 0$ is given , u, v are controls satisfying $|u| \le 1,$ and $|v| \le 1$ conditions.

Remark3. *This example has borrowed from the article[1] by Zakharov. K. In this article author first, investigate necessary optimality conditions by using no intersection of the sets of admissible and sets of decreasing direction of the minimizing function for the problem data. In mentioned article , author used restriction and switching conditions. At the end of the article he gained two nonlinear equations as follows*

$$\tau^2 + 2\tau\left[1 - e^{\tau - T}\right] - 2c = 0\,,\ \tau - T - e^{\tau - T} = 0$$

and added that by solving this system equation we can find switching condition τ *and no fixed endpoint* T *.It means that in the article [1], author get necessary optimality conditions for the switching condition, but at the end of the article he illustrate one example which by using its necessary conditions he cant get find switching points by using its necessary conditions. He applied for the nonlinear system equations which some time without numerical methods it cant be solved. But in the above mentioned two step method, without the switching conditions it can be find switching instants and control, for example, by using the methods of gradient projection .Main difference between article [1] and the presented article are without using switching conditions the above mentioned problem, it can be can be solved presented problem .*

References

1. Zakharov, K., "Optimization of step control systems," *Autom. Remote Control 42 (1982)*.
2. Boltyanskii, V., "The maximum principle for variable structure system," *International Journal of Control vol.77, no.17, pp. 1445-1451* (*2004*).
3. Magerramov, Sh. F. and K.B Mansimov, "Optimization of a class of discrete step control systems," (*Russian, English*) - *Comput. Math. Math.Phys. 41(2001)3, pp. 334-339. Translation from Zh.Vychisl Mat. Mat. Fiz. 41, No.3, pp. 360-366* (*2001*).
4. Shahlar F.Maharramov, 2010 "Necessary optimality conditions for switching control problem" *American Institute of mathematical sciencese, Journal of industrial and manegement optimization, vol.6, No 1, pp. 47-57.*
5. Shahlar F.Maharramov., Exhausters and Quasidifferential in switching control problem, *Submitted paper progress in Sspringer, Optimization, 10 p.*
6. Sorin C. Bengea. and DeCarlo A.Roymand, "Optimal control of switching stystem," Automatica vol.41 (1): pp. 11-27 (2005).
7. Sussmann, H.J., "Set valued differentials and the hybrid maximum principle," EEE Conference on Decision and Control, pp. 3972-3977 (1999).
8. Maharramov, Sh.F., " Optimality condition of a no smooth switching control System," *Automotic control and Computer Science, vol.42.no.2, pp. 94-101* (*2008*).
9. Kharatishvili. G.L and Tadumadze. T., Variation formulas of solutions and optimal control problems for differential equations with retarded argument. J. Math. Sci, (NY), 104(1)(2007), 1-175.
10. Dmitruk, A.V. and A.M. Kaganovich, "*The Hybrid Maximum Principle is a Consequence of Pontryagin Maximum Principle*," System and Control Letters.,Volume 57, Issue 11, pp. 964-970 (2008).
11. Xu, X.P and P.J. Antsaklis, "A dynamic programming approach for optimal control of switched system," Proc.IEEE Conf. Decision Control, pp.1822-1827 (2000).