STABILITY LOSS OF THE MICRO/NANO-FIBER IN THE VISCOELASTIC MATRIX NEAR THE FREE CONVEX CYLINDRICAL SURFACE

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In the paper [1] the solution method for problems related to the micro/nano mechanics of a periodically curved fiber near a convex cylindrical surface was proposed and applied. Note that within the scope of the continuum mechanics the micro and nano fibers are determined according to the ratio of the modules of elasticity $E^{(f)}/E^{(m)}$, where $E^{(f)}(E^{(m)})$ is the modulus of elasticity of a fiber (matrix0 material: if $300 \le E^{(f)}/E^{(m)} \le 1000 (E^{(f)}/E^{(m)} < 300)$ the composite is considered as polymer matrix+nano-fiber (polymer matrix+micro-fiber). According to the foregoing statement, in the present work the stability loss of the micro/nano fiber near the convex free cylindrical surface is studied by the use of the method developed in the paper [1].

It is assumed that the region occupied by the matrix which contains the fiber is $\{0 \le r < R,\ 0 \le \theta \le 2\pi,\ -\infty < z < +\infty\}$. Moreover, it is assumed that the fiber has infinitesimal waviness along its length and the fibers cross section which is perpendicular to its middle line tangent vector, is a circle with constant radius R_0. In the natural state, we associate the Langrangian cylindrical system of coordinates $O_r\theta_z$ $(O_o r_0 \theta_0 z_0)$ and the Cartesian system of coordinates $Ox_1x_2x_3$ $(O_0 x_{10} x_{20} x_{30})$ with the cylinder (the fiber). Between these coordinates the following relations are satisfied. $x_2 = x_{20}$, $x_3 = x_{30}$, $z = z_0$, $re^{i\theta} = R_{10} + r_0 e^{i\theta_0}$. The middle line of the fiber is given by the equation $x_{30} = \eta,\quad x_{10} = A\sin(2\pi\eta/\ell)\cos\beta,\quad x_{20} = A\sin(2\pi\eta/\ell)\sin\beta,$ where η is a parameter and $\eta \in (-\infty, +\infty)$, A is the amplitude of the curving form, ℓ is the wave length of the curving form and β is the angle between the plane $O_0 x_{10} x_{30}$ and the plane on which the middle line of the fiber lies. Suppose that $A << \ell$, we introduce the small parameter $\varepsilon = A/\ell,\quad 0 \le \varepsilon << 1.0$.

Below the values related the cylinder and the fiber will be denoted by upper indices (1) and (0). Assume that the fiber and the surrounding cylinder (matrix) materials are isotropic and homogeneous. Moreover we assume that the material of the fiber is elastic, but the material of the matrix is viscoelastic. Within the scope of the piecewise homogeneous body model with the use of the three-dimensional geometrically non-linear exact equations of the theory of visco-elasticity we investigate the development of the fibers initial infinitesimal imperfection in the case where the body is loaded (compressed) at infinity by uniformly distributed normal forces with an intensity p acting in the direction of the Oz axis. For this purpose we write the field equations which are satisfied within the fiber and surrounded cylinder separately. Note that under writing below the mentioned equations we use conventional tensor notation.

$$\nabla_i \left[\sigma^{(k)in} (g_n^j + \Delta_n u^{(k)j}) \right] = 0,\quad 2\varepsilon_{ij}^{(k)} = \nabla_j u_m^{(k)} + \nabla_m u_j^{(k)} + \nabla_j u^{(k)n} \nabla_m n_n^{(k)},$$

$$\sigma_{(m)}^{(k)} = \lambda^{(k)} (e^{(k)} \delta_i^n) + 2\mu^{(k)*} \varepsilon_{(in)}^{(k)},\quad e^{(k)} = \varepsilon_{rr}^{(k)} + \varepsilon_{\theta\theta}^{(k)} + \varepsilon_{zz}^{(k)}.$$

$$\lambda^{(k)*}\varphi = \lambda^{(k)0}\varphi(t) + \int_0^t \lambda^{(k)}(t-\tau)\varphi(\tau)d\tau, \quad \mu^{(k)*}\varphi = \mu^{(k)0}\varphi(t) + \int_0^t \mu^{(k)}(t-\tau)\varphi(\tau)d\tau \quad (1)$$

Assume that on the interface between the fiber and surrounding medium (denote this surface by S_0) the complete contact conditions are satisfied.

$$\left[\sigma^{(1)in}(g_n^j + \Delta_n u^{(k)j}\right]_{s_0} n_{oj} = \left[\sigma^{(0)in}(g_n^j + \Delta_n u^{(k)j})\right]_{s_0} n_{0j}, \qquad u_{(j)}^{(1)}\Big|_{s_0} = u_{(j)}^{(0)}\Big|_{s_0}. \quad (2)$$

Moreover on the cylindrical surface $r = R$ the following conditions are also satisfied.

$$\left[\sigma^{(1)in}(g_n^j + \Delta_n u^{(k)j})\right]\Big|_{r=R} n_j = 0, \quad (3)$$

Where $n_{oj,}\ n_j$ are the components of the unite normal vector to the surface S_0 and $r = R,$ respectively.

Using the bucking criterion $\left|u_r^{(0)}\right| \to \infty$ as $t \to t_{cr}$, the numerical results are presented and discussed Note that these results are presented for the case where the material of the fiber is pure elastic, but the material of the matrix is viscoelastic. The viscosity of the matrix material is described by the fractional-exponential operator by Rabotnov.

References

[1] Akbarov S.D., Mamedov A.R. On the solution method for problems related to the micro-mechanics of a periodically curved fiber near a convex cylindrical surface. CMES: Computer modeling in Engineering and Sciences. Vol. 42(3), pp. 257-296.