ON A CONTROL PROBLEM FOR HEAT EXCHANGE PROCESS

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In the work, we consider a control problem for heating a thermofor in a heating apparatus, which delivers heat into a closed heat supplying system, placed into a heating stove.

At certain points of the heating apparatus, we place sensors fro measuring the temperature of the thermofor, the values of which are used for assigning the quantity of heat delivered into the heating stove.

The heating process of the thermofor in the heating stove of the heating apparatus can be described by the following transport equation $[1,2]$:

$$\frac{\partial U}{\partial t}+a\frac{\partial U}{\partial x}=\alpha\left[U(x,t)-\upsilon(t)\right], x\in[0,l], t\in[0,T], \tag{1}$$

where $U=U(x,t)$ is the temperature of the thermofor at the point x of the heating apparatus at the moment of time t; l is the length of the tube of the heating apparatus, in which the thermofor is heated; a is the movement velocity of the thermofor in the heat supplying system, the value of which is constant for all points of the heat supplying system, i.e. the movement is assumed to be steady-state (stationary); α is given value of heat exchange coefficient between the heating stove and thermofor in the heating apparatus; $\upsilon(t)$ is the temperature inside the heating stove, by means of which we control the heating process of the thermofor, satisfying the following technological constraint:

$$\underline{\upsilon}\le\upsilon(t)\le\overline{\upsilon}\,. \tag{2}$$

Let L be the length of the heat supplying system, and the thermofor being heated in the heating stove requires the time $T^{retard}=L/a$ for returning to the beginning of the stove, i.e.

$$U(0,t)=U\left(l,t-T^{retard}\right)\xi(t)\,. \tag{3}$$

The function $\xi(t)$ defining heat loss in the movement process in the heating system, reasoning from practical considerations, satisfies the evident condition:

$$0<\underline{\xi}\le\xi(t)\le 1,\ t\in[0,T]. \tag{4}$$

It is assumed to be smooth (differentiable), at that

$$\left|\xi'(t)\right|\le k,\ t\in[0,t], \tag{5}$$

where k is given quantity.

Heat loss in the heat supplying system, determined by the function $\xi(t)$, satisfying conditions (3) and (4), is unknown a priori, and depends on various external uncontrollable factors. Denote the set of heat loss functions, satisfying (3) and (4), by $K(\xi)$, assuming that the density function $P(\xi(t))$ is given on this set.

The control problem for the heating process of the thermofor consists in the necessity of keeping such temperature of the heating stove that provides certain temperature U^* for the thermofor at the output of the heating stove in the presence of all possible admissible values of heat loss $\xi(t)$.

The corresponding control criterion is given as follows:

$$J(\upsilon)=\int_{K(\xi)}I(\upsilon;\xi)dP\left(\xi(t)\right)+\varepsilon\left\|\upsilon(t)\right\|^2_{L^2[0,T]}, \tag{6}$$

$$I(\upsilon,\xi)=\int_0^T\left[U\left(l,t;\xi(t),\upsilon(t)\right)-U^*\right]^2 dt\,.$$

Suppose that several sensors are placed at arbitrary n points $x_i\in[0,l],\ i=1,...,n$ of the heating apparatus. At these points, we can continuously measure the temperature $[2]$:

$$U_i(t)=U(x_i,t),\ t\in[0,T],\qquad(7)$$

or measure the temperature only at discrete moments of time

$$U_{ij}=U(x_i,t_j),\ t_j\in[0,T],\ j=1,...,m\,.\qquad(8)$$

Particularly, the moments of time when we take measurements can be uniform:

$$t_j=j\Delta\,,\ j=0,1,...N_t\,,\ N_t=[T/\Delta].\qquad(9)$$

Here $[a]$ designates the integer part of number a .

Consider the following modification of the temperature control system in order to build the control system for the heating stove with continuous feedback (8):

$$\upsilon(t)=\upsilon(t,V,\gamma)=\int_{t-T^{retard}}^{t}V(\tau)\sum_{i=1}^{n}\gamma_iU(x_i,\tau)d\tau\,,\qquad(10)$$

where $\gamma_i=const$ are weighting coefficients defining the importance of a measurement at the points $x_i,i=1,...,n$:

$$\gamma\in\Gamma=\left\{\gamma\in E^n:0\le\gamma_i\le1,\ i=1,...,n,\ \sum_{i=1}^{n}\gamma_i=1\right\}.\qquad(11)$$

It is seen from (8) that the temperature of the heating stove at the current moment of time is determined by the measured values only for the time interval $\left[t-T^{retard},t\right]$, during which the thermofor having been heated in the heating stove passes closed heat exchange contour and returns to the heating stove.

In case if measurements are taken only at discrete moments of time at the points x_i; $i=1,..n$, the following modification of temperature control in the heating stove is assumed:

$$\upsilon(t)=\upsilon(t;V,\gamma)=\sum_{j=N_t-N^{retard}}^{N_t}V_j\sum_{i=1}^{n}\gamma_iU(x_i,t_i),\ t\in\left[(N_t-1)\Delta;N_t\Delta\right].\qquad(12)$$

Thus, in this case, control by the heating stove is implemented on a class of piecewise constant functions; from (11), it follows that the temperature in the heating stove at the intervals of time between the measurements of temperature at the points $x_i,\ i=1,...,n$ is established according to the last N^{retard} measurements obtained at the interval of time $\left[t-T^{retard},t\right]$.

Investigate the case when the control has form (10). If we take into account (10) in (1), then we obtain:

$$\frac{\partial U(x,t)}{\partial t}+a\frac{\partial U(x,t)}{\partial x}=\alpha\left[U(x;t)-\sum_{i=1}^{n}\gamma_i\int_{t-T^{retard}}^{t}V(\tau-t+T^{retard})U(x_i,\tau)d\tau\right].\qquad(13)$$

Thus, with respect to control process for heating the thermofor with feedback, we obtain an optimal control problem described by loaded integral-and-differential equation (13) [4, 5] with boundary conditions with retarded argument (3). The function $V(t),\ t\in\left[0,T^{retard}\right]$, satisfying (2), and weighting coefficients $\gamma_i,\ i=1,...,n$, satisfying (11), are optimized in the problem.

To solve optimal control problem (13), (11), (2)-(6), we use method of penalty functions to take into account constraints (2), considering the sequential minimization of the penalty functional:

$$S(V,\gamma) = \int_{K(\xi)} I(V,\gamma;\xi)dP(\xi(t)) + \varepsilon \|V(t)\|^2_{L^2[0,T^{retard}]} +$$

$$+ R\int_0^T \left\{ [\min(\upsilon(t;V,\gamma) - \underline{\upsilon};0)]^2 + [\min(\overline{\upsilon} - \upsilon(t;V,\gamma);0)]^2 \right\} d\tau,$$

when $R \to +\infty$. To take into account constraints (11), we use iterative gradient projection method:

$$\begin{pmatrix} V(t) \\ \gamma \end{pmatrix}^{\nu+1} = P_\Gamma \left[\begin{pmatrix} V(t) \\ \gamma \end{pmatrix}^{\nu} - \alpha \begin{pmatrix} \dfrac{\partial S(V^\nu,\gamma^\nu)}{\partial V} \\ \dfrac{\partial S(V^\nu,\gamma^\nu)}{\partial \gamma} \end{pmatrix} \right], \nu = 0,1,..., \quad (14)$$

where $P_\Gamma(\gamma)$ is the projection operator of the vector γ on the set Γ from (11), which has comparatively simple construction.

To build procedure (14), we obtain formulas for the gradient of the functional on the optimized parameters:

$$\nabla S(V,\gamma) = (\nabla_V S(V,\gamma), \nabla_\gamma S(V,\gamma))^T.$$

These formulas allow to formulate necessary optimality conditions. On the other hand, they are used to obtain the numerical solution to the problem stated.

References

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