ADAPTIVE SYSTEM OF IDENTIFICATION AND CONTROL NONLINEAR STOCHASTIC DYNAMIC OBJECTS IN A CLASS OF MODELS WIENER

Givi Bolkvadze

Institute of Cybernetics, Tbilisi, Georgia
geogivi@mail.ru

The urgency of a problem of working out of new methods of modeling and control of technical systems (TS) and technological processes (TP) is caused by necessity of improvement of quality of functioning of these systems for the purpose of achievement of release of high-quality end-products, decrease energy - and labor inputs of the TS and TP, automation of processes of gathering and data processing about system, processes of identification and adaptive control on the basis of modern information technologies. The big attention is given to synthesis **of adaptive systems of identification and control (AdSIC)** where information on object and conditions of its functioning are not required full aprioristic. Creation effective **AdSIC** is usually connected by such objects with modeling and identification processes. Increase of requirements to efficiency **AdSIC** involves increase of requirements to accuracy and adequacy of models of objects.

In this paper for the **nonlinear stochastic dynamic objects of control (NSDOC)** is considered problems of construction **AdSIC on the basis of personal computers**. On a basis the statistical an output/input, removed at normal functioning NSDOC, Wiener's nonlinear dynamic model [1] which to enter in a class of Bloch no-focused models [2] is constructed. The structure of model of Wiener is represented as consecutive connection of linear dynamic blocks and nonlinear static elements (structural identification). After definition of structure of model in the opened contour of control the identification problem is reduced to a problem, which essence consists in minimization of criterion of quality of identification on parameters, where as algorithm of identification the optimum **two-stage recurrent identification algorithm (TSRIA)** on the basis of **an average method of the least squares (AMLS)** (parametrical identification) [1,4] is applied. The model of Wiener constructed thus is applied in the identification closed a contour and controls at creation **AdSIC** to tracking for wished value of output NSDOC by means of TSRIA, AMLS [4] and **adaptive algorithm of control (AdAC)** [4]. By means of computer modeling is confirmed effective functioning constructed **AdSIC**.

Construction of model of Wiener in an open contour of control is carried out so. As object of control it is considered NSDOC in the conditions of which normal functioning on an input is observed operating process $X(t) \in R^1$, and on an output - operated process $Y(t) \in R^1$ which are assumed stationary and permanently connected in dispersive sense aligned ergotikal processes in (Ω, F, P) space. The discrete moments of time $n = 1, 2, \cdots, N, \cdots$ are measured input/output data as $\{x_n, y_n\}$. Sequence

$$\{x_n, y_n\}_{n=1}^{\infty} \tag{1}$$

represents infinite sample statistically independent supervision casual $X(t)$ and $Y(t)$ processes. on the basis of realization (1) the model in which as consecutive connection following subsystems are included is under construction:

1. The linear dynamic block of model of Wiener

$$y_{1,k} = \sum_{i=1}^{m} g(i) y_{k-i} + \sum_{i=1}^{l} h(i) x_{k+1-i} = g^T \mathbf{y}(k-1) + h^T \mathbf{x}(k), \quad k = l+1, l+2, \cdots, N, \cdots,$$
$$\mathbf{y}(k-1) = [y_{k-1}, \cdots, y_{k-m}]^T \in R^m, \quad \mathbf{x}(k) = [x_k, \cdots, x_{k-l+1}]^T \in R^l, \tag{2}$$

where $g = [g(1), \cdots, g(m)]^T \in R^m, h = [h(1), \cdots, h(l)]^T \in R^l$ - a vector of unknown weight factors of the linear block of model of Wiener.

2. The nonlinear static element of model of Wiener is realized by an eminence in a square

$$y_{2,k} = y_{1,k}^2 = [\sum_{i=1}^{m} g(i) y_{k-i} + \sum_{i=1}^{l} h(i) x_{k+1-i}]^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} g(i) g(j) y_{k-i} y_{k-j} +$$
$$\sum_{i=1}^{m}\sum_{j=1}^{l} g(i) h(j) y_{k-i} x_{k+1-j} + \sum_{i=1}^{l}\sum_{j=1}^{l} h(i) h(j) x_{k+1-i} x_{k+1-j} = z_k^T \theta. \quad (3)$$

Definitively, Wiener's model becomes

$$\tilde{y}_k = y_{2,k} + \eta_k = z_k^T \theta + \eta_k, k = l+1, l+2, \cdots, N, \cdots; z_k = [z_{1,k}^T; z_{2,k}^T; z_{3,k}^T]^T \in R^d, d = mm + ml + ll, \quad (4)$$

$$z_{1,k} = \mathbf{y}(k-1)\mathbf{y}^T(k-1) = \begin{bmatrix} y_{k-1}y_{k-1}, \cdots, y_{k-1}y_{k-m} \\ \vdots \\ y_{k-m}y_{k-1}, \cdots, y_{k-m}y_{k-m} \end{bmatrix} = [z_1(k), \cdots, z_m(k); \cdots, z_{mm}(k)]^T \in R^{mm},$$

$$z_{2,k} = \mathbf{y}(k-1)\mathbf{x}^T(k) = \begin{bmatrix} y_{k-1}x_k, \cdots, y_{k-1}x_{k-l+1} \\ \vdots \\ y_{k-m}x_k, \cdots, y_{k-m}x_{k-l+1} \end{bmatrix} = [z_{mm+1}(k), \cdots, z_{mm+l}(k); \cdots, z_{mm+ml}(k)]^T \in R^{ml},$$

$$z_{3,k} = \mathbf{x}(k)\mathbf{x}^T(k) = \begin{bmatrix} x_k x_k, \cdots, x_k x_{k-l+1} \\ \vdots \\ x_{k-l+1}x_k, \cdots, x_{k-l+1}x_{k-l+1} \end{bmatrix} = [z_{mm+ml+1}(k), \cdots, z_{mm+ml+l}(k); \cdots, z_{mm+ml+ll}(k)]^T \in R^{ll}.$$

Coordinates of vectors θ are product of components of vectors g, h and on itself, and represent values of unknown factors of each input of model (4)

$$\theta = [(\theta^{(1)})^T; (\theta^{(2)})^T; (\theta^{(3)})^T]^T \in R^d, \quad (5)$$

$$\theta^{(1)} = \mathrm{gg}^T = \begin{bmatrix} g(1)g(1), \cdots, g(1)g(m) \\ \vdots \\ g(m)g(1), \cdots, g(m)g(m) \end{bmatrix} = [\theta_1, \cdots, \theta_m, \theta_{m+1} \cdots \theta_{2m}, \cdots, \theta_{(m-1)m+1}, \cdots, \theta_{mm}]^T,$$

$$\theta^{(2)} = \mathrm{gh}^T = \begin{bmatrix} g(1)h(1), g(1)h(2), \cdots, g(1)h(l) \\ \vdots \\ g(m)h(1), g(m)h(2), \cdots, g(m)h(l) \end{bmatrix} = [\theta_{mm+1}, \cdots, \theta_{mm+l}, \theta_{mm+l+1} \cdots \theta_{mm+2l}, \cdots, \theta_{mm+ml}]^T,$$

$$\theta^{(3)} = \mathrm{hh}^T = \begin{bmatrix} h(1)h(1), h(1)h(2), \cdots, h(1)h(l) \\ \vdots \\ h(l)h(1), h(l)h(2), \cdots, h(l)h(l) \end{bmatrix} = [\theta_{mm+ml+1}, \cdots, \theta_{mm+ml+l}, \theta_{mm+ll+l+1}, \cdots, \theta_{mm+ml+ll}]^T$$

and which are required to be defined. In system (4); η_k - a hindrance on an output a zero average, final dispersion and not correlated both with an input, and with an output.

The problem of definition θ dares as a problem of recurrent identification. For this purpose. The criterion of quality of identification is entered

$$J^{(N)}(\theta) = M_N[y_k - \hat{y}_k]^2 = \frac{1}{N-l-1}\sum_{k=l+1}^{N}[y_k - z_k^T\theta]^2 = \frac{1}{N-l-1}\|Y_N - Z_N\theta\|_2^2, \; N = l+1, \cdots, \quad (6)$$

Where $Y_N = [y_{l+1}, \cdots, y_N]^T \in R^T$, $Z_N = [z_{l+1}, \cdots, z_N]^T \in M_{(N-l)\times(m+l)}$, M_N estimations of a population mean on the basis of (1) statistical with a length N. The problem of recurrent identification is reduced to definition of such optimum value of a vector, θ_N for which at everyone N the estimation (6) reaches a minimum:

$$\theta_N = \arg \min_{\theta_N \in R^{m+l}} J^{(N)} \quad (7)$$

For definition of a point of a minimum (7) from a condition of a minimalist (6) the following system of the equations of dispersive identification turns out

$$K_{YZ}^{(N)} = K_{ZZ}^{(N)}\theta, \quad (8)$$

where $K_{YZ}^{(N)}$ - a vector of an estimation of mutually correlation functions of a vector of supervision Y_N and matrixes of supervision Z_N, and $K_{ZZ}^{(N)}$ - a matrix of an estimation of correlation function of matrixes of supervision Z_N. For the decision of the equation (8) concerning unknown vector θ are under construction TSRIA [2] which is realized in three stages. **Step 1.** Vector θ is estimated on the basis of AMLS in a kind

$$\theta_N = \theta_{N-1} + \mu r_N^{-1}[y_N - z_N^T \theta_{N-1}] z_N,\ N = l+2, \cdots,\ r_N = [\sum_{k=l+1}^{N} z_k z_k^T]/(N-l-1), \tag{9}$$

where μ - adaptation factor, $0 < \mu < 2$. **Step 2.** Matrixes are formed of vector θ_N

$$\Theta_{gg}(N) = \begin{bmatrix} \theta_{1,N}, \cdots, \theta_{m,N} \\ \vdots \\ \theta_{(m-1)m+1,N}, \cdots, \theta_{mm,N} \end{bmatrix}, \Theta_{gh}(N) = \begin{bmatrix} \theta_{mm+1,N}, \cdots, \theta_{mm+l,N} \\ \vdots \\ \theta_{mm+(l-1)l+1,N}, \cdots, \theta_{mm+ml,N} \end{bmatrix},$$
$$\Theta_{hh}(N) = \begin{bmatrix} \theta_{mm+m+1,N}, \cdots, \theta_{mm+ml+l,N} \\ \vdots \\ \theta_{mm+ml+(l-1)l+1,N}, \cdots, \theta_{mm+ml+ll,N} \end{bmatrix}. \tag{10}$$

Matrixes (10) are estimations of a matrix (5). We will consider singular decomposition of matrix $\Theta_{gh}(N)$ for the purpose of reception of separate estimations for vectors g and h [2] which looks like [3]

$$\Theta_{gh}(N) = \sum_{i=1}^{q} \sigma_{i,N} \mu_{i,N} v_{i,N}^T, \quad q = \min(m,l), \tag{11}$$

Where $\mu_{i,N} = [\mu_{i,1,N}, \cdots, \mu_{i,m,N}]^T \in R^m,\ v_{1,N} = [v_{1,1,N}, \cdots, v_{1,l,N}]^T \in R^l,\ i = \overline{1,q}$

Are accordingly m and l- dimensional orthonormal vectors, and $\sigma_{i,N},\ i = \overline{1,q}$ singular numbers of matrix $\Theta_{gh}(N)$. **Step 3.** Let s_μ designates a sign on first nonzero element $\mu_{1,N}$. We will define for separate co-ordinates of vectors of weight factors g, h estimations thus

$$\begin{aligned} &\tilde{g}_N = [\tilde{g}_N(1), \cdots, \tilde{g}_N(m)]^T = s_\mu \mu_{1,N}; \quad \mu_{1,N} = [\mu_{1,1.N}, \cdots, \mu_{1,m,N}]^T \in R^m; \\ &\tilde{g}_N(1) = s_\mu \mu_{1,1,N}; \cdots, \tilde{g}_N(m) = s_\mu \mu_{1,m,N}; \\ &\tilde{h}_N = [\tilde{h}_N(1), \cdots, \tilde{h}_N(l)]^T = s_\mu \sigma_{1,N} v_{1,N}; \quad v_{1,N} = [v_{1,1,N}, \cdots, v_{1,l,N}]^T \in R^l; \\ &\tilde{h}_N(1) = s_\mu \sigma_{1,N} v_{1,1,N}; \cdots, \tilde{h}_N(l) = s_\mu \sigma_{1,N} v_{1,l,N}. \end{aligned} \tag{12}$$

Than
$$\left\| \begin{matrix} [\tilde{g}_N(1)\tilde{g}_N^T,, \cdots, \tilde{g}_N(m)\tilde{g}_N^T; \tilde{g}_N(1)\tilde{h}_N^T, \cdots, \tilde{g}_N(m)\tilde{h}_N^T; \\ \tilde{h}_N(1)\tilde{h}_N^T, \cdots, \tilde{h}_N(l)\tilde{h}_N^T]^T - \theta_N \end{matrix} \right\|_2 = \left\| \begin{bmatrix} vec(\tilde{g}_N \tilde{g}_N^T) \\ vec(\tilde{g}_N \tilde{h}_N^T) \\ vec(\tilde{h}_N \tilde{h}_N^T) \end{bmatrix} - \theta_N \right\|_2 =$$
$$= \left\| \tilde{g}_N \tilde{g}_N^T - \Theta_{gg}(N) \right\|_F + \left\| \tilde{g}_N \tilde{h}_N^T - \Theta_{gh}(N) \right\|_F + \left\| \tilde{h}_N \tilde{h}_N^T - \Theta_{hh}(N) \right\|_F, \tag{13}$$

where $\|\cdot\|_F$ stands for the matrix Frobenius norm [3, 4]. Thus, that estimations (12) satisfied to a condition (13), besides, estimations (10), (11) should satisfy for (13) conditions

$$[\mu_{1,N}; \quad \sigma_{1,N} v_{1,N}] = \underset{w \in R^m, v \in R^l}{\arg\min} \left\| \Theta_{gh}(N) - wv^T \right\|_F^2. \tag{14}$$

The decision of a problem of minimization (14) rather $[\mu_{1,N}; \quad \sigma_{1,N} v_{1,N}]$ is provided by the singular value decompositions (SVD) of matrix $\Theta_{gh}(N)$ under the formula (11) and lemmas

A.1 from [4]. Convergence $\theta_N \to \theta$ the river of century a.s. (almost surly) or $\Theta_{gg}(N) \to \Theta_{gg}$, $\Theta_{gh}(N) \to \Theta_{gh}$, $\Theta_{hh}(N) \to \Theta_{hh}$ theorems 2.1 of [4] are proved similarly.

At the problem decision adaptive control with parametrical identification in the closed contour as model is considered **on p - steps Wiener's advancing model** in such kind

$$\tilde{y}_{n+p} = z_n^T \theta + \beta u_n + \rho_{n+p},\ n = N_1 + l + 1, \cdots, \tag{15}$$

where z_n - the entrance vector of model (15), which components are defined under the formula (4). Number N_1 - is the discrete moment of time or an iteration step on which identification process in the opened contour comes to the end; u_n - an operating input; β - control factor; ρ_{n+p} - a hindrance sliding on the average are defined similarly from [5]. Let sequence y_n^* the item of century of the limited random variables of output NSDOC wished value and y_{n+p}^* be F_n - measurable. We will define **criterion of control** in a kind

$$J(u_n) = M\{\left|\tilde{y}_{n+p} - y_{n+p}^*\right|^2 \mid F_n\} + \lambda u_n^2,\ \lambda \geq 0;\ n = N_1 + l + 1, N_1 + l + 2, \cdots. \tag{16}$$

The problem of adaptive control in this case consists in definition of such optimum control u_n^*, which satisfies to a condition

$$u_n^* = \arg \min_{u_n \in R^1} J(u_n) \tag{17}$$

From a condition of a minimalists (17) the optimum control formula turns out

$$u_n^* = (\beta^2 + \lambda)^{-1} \beta [y_{n+p}^* - z_n^T \theta],\ n = N_1 + l + 1, N_1 + l + 2, \cdots \tag{18}$$

also is **on p - steps advancing optimum control** [5].

For an estimation of vector θ from (18), defined on (5), it is necessary to solve a problem of parametrical identification in the closed contour of control. Are with that end in view applied TSRIA, on p - steps advancing algorithm AMLS in a kind

$$\begin{aligned} r_n^{-1} \theta_{n+p} &= r_{n-1}^{-1} \theta_{n+p-1} + [\tilde{y}_{n+p} - (\beta^2 + \lambda)^{-1} \beta^2 (y_{n+p}^* - z_n^T \theta_n)] z_n, \\ r_n &= r_{n-1} - a_n r_{n-1} z_n z_n^T r_{n-1}, r_{N_1} > 0, \quad a_n = (1 - z_n^T r_{n-1} z_n)^{-1}. \end{aligned} \tag{19}$$

Where the operating input of model (15) is estimated **on adaptive by algorithm of control - AdAC** in a kind

$$\tilde{u}_n = (\beta^2 + \lambda)^{-1} \beta [y_{n+p}^* - z_n^T \theta_n] \tag{20}$$

Researches of questions of convergence of algorithms (20), AMLS under formulas (9) and (19) are carried out similarly from [5]. AdSIC it is possible to apply successfully in management of the TS and TP metallurgical, heat power, petrochemical and other manufactures, and also in the easy and food industries.

References

1. Bolkvadze G.R. Wieners models in problems of recurrent identification // Proceedings of the IV International conference «System identification and control problems» SICPRO `05. Moscow, 25-28 January, 2005, pp. 300 - 311.
2. Haber R. and Unbehauen H. Structure identification of nonlinear dynamic systems – a survey on input / output approaches // Automatica. Vol. 26. 1990. № 4, pp. 651 - 677.
3. Horn R.A., Johnson C.R. Matrix Analysis. Cambridge: Cambridge Univ. Press, 1985.
4. Bai E.W. An optimal two-stage identification algorithm for Hammerstein - Wiener nonlinear systems // Automatica, 1998, vol. 34, No. 3, pp. 333 - 338.
5. Luo G. Optimal adaptive tracking for stochastic systems//Proceeding of 14the World Congress IFAC, 1999, pp. 385 - 390.