A MODEL TO CONTROL A QUEUE IN A VOICE SELF-SERVICE PORTAL WITH FAST AND SLOW SERVERS

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In this paper, we consider mathematical models for analysis and optimization of call centers of modern architecture having a self-service facility on the base of computer speech recognition. In such centers, service is provided by both operators and self-service ports. It is shown that the optimal policy for queue control improves service characteristics.

1. Introduction

Improvement of characteristics of service is an actual problem for the call centers. In [1-3], the models applied to the description of functioning of the call service centers on the basis of the "classical" queueing system theory are investigated. Models in [3-8] deal with the factors such as multi-class calls, various service disciplines, "patience" of a user, repeated calls.

However the contact processing centers of the modern architecture with voice selfservice systems based on speech technologies are not investigated. Such centers allow to providing the full service of the majority of incoming calls in automatic mode, without participation of the operators. An example of the call center processing with self-service systems on the basis of speech technologies is the system "Automatic dispatcher" for a taxi service, system "Autosecretary" for scheduling the calls, etc. [9-12]. Customer calls arrive in the call processing centre from different type networks: a public switched telephone network, networks of mobile communication operators, the Internet. The call centre includes ports of self-service on the basis of computer speech recognition and an operator group. The calls are directed by different algorithms. One of the widely-used algorithms implies that incoming calls go the self-service ports first if one of them is idle, and to an operator group, otherwise. The calls which are not served because of the recognition errors of self-service ports are directed to an operator group.

2. Model of the call centre with self-service facility with the limited queue

The call centres with self-service facility can be investigated by means of an open exponential multi-server queueing network with two nodes, node 1 for self-service facility and node 2 for a group of operators. The node *i* represents a multi-server queueing system with n_1 identical servers, i = 1, 2. Let the waiting space be limited for node 1 and unlimited for node 2 [14].

The process of call arrival is the Poisson with parameter λ . The calls are directed to node 1 or 2 depending on the number of idle servers in node 2.

If there are idle servers in node 2, the arriving calls are directed to node 2, otherwise they are directed to node 1. If one of busy servers in node 2 becomes idle and there are no waiting calls in the node, a call in the head of the queue in node 1 (if the queue is not empty), goes to node 2 and occupies the idle server. The call service time in node *i* is exponentially distributed with parameter μ_i .

Also, we assume that in the first node, a call is served successfully with probability 1-p, and leaves the network upon its service completion. Otherwise, with probability p, the service is unsuccessful (the call processing failed, and the call should be reprocessed by an operator), and the call goes to the second node (see Fig. 1).

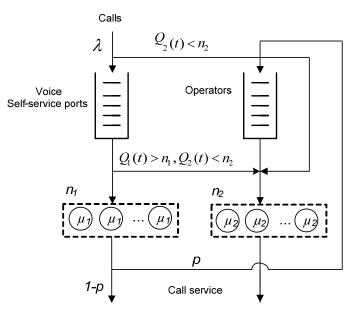


Fig. 1. The scheme of a queueing network with two nodes

The network behaviour is described by the following two-dimensional Markov process: ${X(t)}_{t \ge 0} = {Q_1(t), Q_2(t),}_{t \ge 0}$,

where $Q_i(t)$ is the number of calls in node *i*, *i* = 1, 2. Consider the stationary probabilities:

$$\pi(i,j) = \lim_{t \to \infty} \mathbb{P}[X(t) = (i,j)], i = \overline{0,N}, j \ge 0.$$

Below we describe the algorithm to calculate the stationary probabilities based on the matrix analytical approach to the analysis of the multidimensional Markov chains having at most one numerable components [13].

3. Model of the voice self-service centre with unlimited waiting space in nodes

In [14] the open exponential multi-server service queueing network with unlimited waiting space in nodes is investigated.

It is proved, that the balance equations has the product form solution. Calculation of stationary probabilities allows finding the following performance characteristics:

• The mean number of busy servers in the first and second nodes

$$N_1 = \sum_{i=0}^{n_1} \sum_{j=0}^{\infty} \min\{i, n_1\} \pi(i, j), \qquad N_2 = \sum_{i=0}^{n_1} \sum_{j=0}^{\infty} \min\{j, n_2\} \pi(i, j).$$

• The mean number of busy servers in the network

$$L = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (i+j)\pi(i,j) \, .$$

• The probabilities that an arriving call is directed to node 1 and 2, respectively,

$$\tau_1 = \sum_{i=0}^{\infty} \sum_{j=n_2}^{\infty} \pi(i,j), \qquad \tau_2 = 1 - \tau_1.$$

• The mean waiting time in the network can be derived by the Little's formula:

$$W = \frac{L}{\lambda}$$

• The mean waiting time in node 2:

$$W_{2} = \frac{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} j\pi(i,j)}{\lambda(1-\tau_{1}(1-p))}$$

In the next Section, we deal with the problem to find an optimal number of servers which minimises the cost function involving the fees for a call waiting and service and the fee for an idle servers.

4. Optimum queue control for the call centre with fast and slow devices

For the self-service call centres is characteristic the difference in speeds and costs of working devices of the first and second nodes. The node 1 describing facility with unreliable speech recognition service has low service rate but small expenses for service. The node 2 describing the operator group is reliable, faster and simultaneously more expensive in comparison to node 1. The problem of optimal queue control in such system belongs to the class of optimizations problems for controllable Markovian queueing systems [15-17]. The paper [18] deals with the problem of "the slow device".

With respect to the call centres with self-service facility, the problem is to minimise the mean waiting time in the network. To solve the problem, the communication centre is considered in [19] as an open exponential multi-server queueing network with two nodes, each represented by an $M/M/n_i$ multi-server queueing system with identical servers. The input flow is Poisson with parameter λ . The arriving calls is directed to node 1 or 2 depending on the number of calls in node 2. If the number of calls reaches threshold level $q_2(q_2 > n_2)$, an arriving call goes to node 1, and to node 2, otherwise.

When the number of calls in node 2 decreases to $q_2 - 1$, and there are calls waiting in node 1 the call at the head of the node 1 queue goes to the end of the node 2 queue. The choice of such control policy is explained by that fact that the mean waiting time in node 1 can exceed the mean waiting time in node 2 since servers of node 2 have higher service rate.

First, consider the stochastic process $\{X(t)\}_{t \ge 0} = \{Q_1(t), Q_2(t), \}_{t \ge 0}$ describing the network state at the time *t*, where $Q_i(t)$ is number of calls in node *i* node of, i = 1, 2.

Let *E* be the state space of the process $\{X(t)\}_{t>0}$

$$E = \{ x = (i, j) : i, j \ge 0 \},\$$

where state (i, j) means that node 1 has *i* calls, and node 2 has *j* calls. It is proved that the balance equations has the product form solution. It is shown that stationary probabilities of system states are represented as the function of probability $\pi(0, q_2 + 1)$ which, in turn, can be calculated from the normalization condition.

To find an optimal threshold level q_2 the following structure of penalties is considered: $c_{0,k}$ is the cost for one call waiting in node k per unit time,

 $c_{u,k}$ is the cost of one call processing in node k per unit time.

The problem is to minimise the following cost function

$$V(q_2) := V(\lambda, \mu_1, \mu_2, p, n_1, n_2, q_2) \rightarrow \min_{q_2}$$

which in this case takes the form

$$\overline{V}(q_2) = \sum_{k=1}^2 c_{u,k} \overline{C}_k + \sum_{k=1}^2 c_{0,k} \overline{Q}_k.$$

Fig. 2 shows the dependence of the cost function $V(q_2)$ on the probability error p and the intensity λ of the input flow. It is obvious, that the increasing of parameters p and λ leads to the optimal threshold level q_2 increasing since the network utilization grows and it is more necessary to use node 2 with faster servers.

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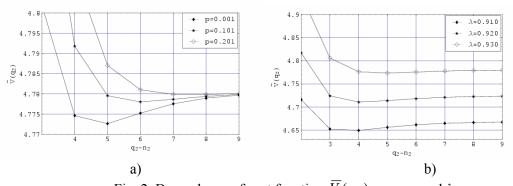


Fig. 2. Dependence of cost function $\overline{V}(q_2)$ on $q_{2,p}$ and λ

5. Conclusions

In the paper, we presented the investigation of the telephone service systems with a combination of traditional methods of service and self-service on the basis of computer speech technologies and a modern interactive handshaking. Models consider the specificity brought by these new technologies. Models can be used for designing of communication centres and service of calls with modern interface technologies.

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