# ON GROWTH RATE OF SOLUTION OF SECOND ORDER NONLINEAR ELLIPTIC EQUATION IN UNBOUNDED DOMAIN

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**Abstract.** This paper deals with quality behavior of the positive Solution $U(x)$ of a non-linear elliptic equation in unbounded domain vanishing zero on the boundary when $|x|$ is sufficiently large depending on the non-linearity character and the geometry of the domain the growth rate of the solution depending on the constant of the elliptic equation and the parameters of the domain is established.

**Keywords:** mathematical physics, equations of elliptic type quality equations, non-divergeaut equation with measurable coefficients.

In the paper, we consider the solution $u(x)$ of the equation of the form… (1), where (2) in an unbounded domain $\Omega = R^n / \cup \varphi$ obtained from $R^n$, $n > \alpha$ by excluding the balls of the same definite radius $\varepsilon$ with centers in the tame shear lattice.

Denote the boundary of domain $\Omega$ by $\partial\Omega = \bigcup_{i_{11},\dots,i_n} \partial B^{\varepsilon}_{i_{11},\dots,i_n}$. It is assumed that the matrix $\|a_{ij}(x)\|$ is symmetric and uniformly positive-definite.

Let $e$ - be an ellipticity on constant of the operator $L$, i.e.

$$e = \sup_{x\in\Omega, |\xi|=1} \frac{\sum_{i=1}^{n} a_{ii(x)}}{\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j}$$

Let the number $S > 0$ satisfy the inequality $s > e - 2$ and $\alpha$ satisfy the inequality $-1 < \alpha < \min\left(1, \frac{2}{s}\right)$

In the paper, we shall use the following simple maximum principle and the growth lemma Maxim $[1, c.15-19]$.

**Maximum principle.** Let $u(x)$ be a positive solution of equation (1), where $\varphi$ $\max_{G} u = \max_{\partial G} u$ satisfies condition (2) in the unbounded domain $G$, continuous in $\overline{G}$. Then $\max_{G} u = \max_{\partial G} u$

**The growth lemma.** Denote by $B_r$ ball in $R^n$ of radius $R$ centered at the origin of coordinates. Let the domain $D \subset B_R$, $R < R_0$, where $R_0$ is sufficiently small, have limit points on the surface $S_R$ of the ball $B_R$ and intersect the ball $B_{P,}$ $\rho = \frac{1}{4}R$

Denote $H = B_p \setminus D$, $\Gamma$ is a part of the boundary of domain $D$ arranged strongly interior to $B_R$.

Let be $u(x)$ a solution of equation (1), where $\varphi(x,u)$ satisfies condition (2), positive in $G$, continuous in $\overline{G}$ and vanishes on $\Gamma$, and H let contain a ball centered at some point $\xi_1$ or radius $\rho_1$ Then $\sup\limits_{x\in G} u(x) \geq (1+\xi\cdot\frac{\rho_1^s}{R^s})\cdot \sup\limits_{x\in G\cap B_\rho} u(x)$, where $\xi$ is a positive constant dependent on $S$.

**Theorem.** Let $u(x)$ be a positive solution of equation (1) in $\Omega$, where $\varphi$ satisfies condition (2). Let $\partial\Omega$ be a boundary of domain $\Omega$ and $u(x)|_{\partial\Omega}=0$; $M(r)=\max\limits_{|x|=r} u(x)$ Then there exist $\beta(s)=const$ such that $M(r)>e^{\beta}\cdot const$, where the constant $\beta$ depends both on $s$ and on $e,n$. (may be M(r)= ∞, with some $r$).

*Proof.* Consider the balls $B_R^0$ and $B_{4R}^0 c$ centered at the origin of coordinates and of radius $R=1/4$ и $4R$, respectively. Since $\varepsilon<1/4$, the set $(R^n\setminus\overline{\Omega})\cap B_R^0$ $R_n$ contains least one ball of radius $\varepsilon$

Applying the growth lemma for the balls $B_R^0$ and $B_{4R}^0$ we get $\sup\limits_{\Omega\cap B_{4R}^0} u(x)\geq\left[1+\xi\cdot\frac{\varepsilon^s}{(1/4)^s}\right]\cdot\sup\limits_{\Omega\cap B_R^0} uu(x)>\left[1+\xi(4\varepsilon)^s\right]\cdot u(0)$ Let the maximum of the function $u(x)$ in the closure of domain $\Omega\cap B_{4R}^0$ be attained at some point $x_1$ on the surface of this ball. Applying the growth lemma once more, we get $\sup\limits_{\Omega\cap B_{4^k\cdot R}^{x_i}} u(x)>\left[1+\xi\cdot(4\varepsilon)^s\right]^2\cdot u(0)$

Denote $r=|x|$. Assume $r=4^k\cdot R$ and apply the growth lemma $k$ times in the indicated formulation. Then we get $\sup\limits_{\Omega\cap B_{4^k\cdot R}^{x_i}} u(x)>\left[1+\xi\cdot(4\varepsilon)^s\right]^k\cdot u(0)$

It follows from the last formula $M(r)>e^{\left[\log_4\frac{r}{R}\right]\cdot\ln(1+\xi(4\xi)^s)}\cdot u(0)$

It follows that of $\beta>0$ is a sufficiently small constant dependent on $s$, then $M(r)>e^{\beta}\cdot u(0)$

Consequently, $M(r)>e^{\beta}\cdot const$.

Remark. Everywhere we considered a positive solution. The case of a negative solution is reduced to the case of a positive solution by changing it's by a contrary one.

### References

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