ANALYSE OF TRAFFIC SYSTEMS BY THE MODELS OF MOVING PARTICLES ON A CIRCLE

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Introduction. The mathematical models of moving particles on an infinite straight line without overtaking have been introduced in [1,2]. In these models a motion of particles depends on a distance between them and each particle can effect the movement of other particles. It was shown that in stationary regime each separately considered particle makes random binomial walk and distribution of the distance between neighbouring particles has been derived. This result was used for calculation of the different characteristics of the road. For instance, the curve, describing dependence the system output capacity on intensity (road density) of particles.

In [3,4] models consisting two moving particles on a circle have been investigated. It was proved that each separately observable particle makes binomial random walk with the same parameters ($r, l = 1 - r$).

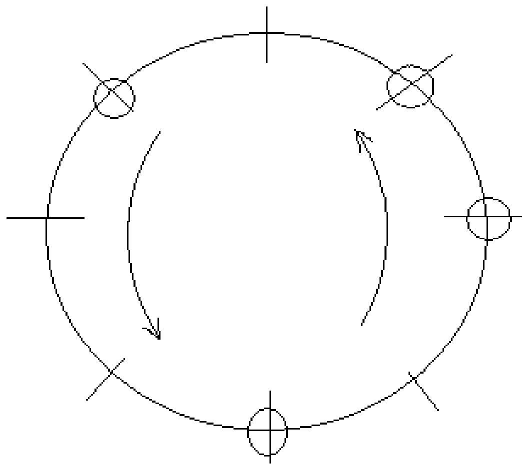

fig. 1

Mathematical models with moving of many particles have complicated structure and their analytical research faces with a lot of troubles, hence for their investigation it is necessary to develop the new approaches of investigations. In [5] some models of moving particles on a closed curve with different rules of motion have been considered, where the fact about random binomial walk of separately considered particle has been derived.

In the present paper the models with many particles on a circle without overtaking are researched by computer simulation.

Problem statement. The motion of particles on the fixed points of a circle is considered. It is assumed that the system changes states at the discrete instants $0, h, 2h, \ldots (h > 0)$. Assume that there are N equidistant points on a circle and s moving particles $(s < N)$. The particles move counter clockwise (fig. 1).

Introduce the following notations:

$\xi_{i,t}$ is a coordinate of the i–th particle at the instant t. Then

$$\rho_{i,t} = \xi_{i+1,t} - \xi_{i,t} \quad (i = 1, \ldots, s-1); \; \rho_{s,t} = \xi_{1,t} - \xi_{s,t};$$

is a distance between a neighbour particles at the instant t.

$\varepsilon_{i,t} = \xi_{i,t+h} - \xi_{i,t}$ defines a motion of the i–th particle at the instant t, i.e. it takes value, 1 if particle makes a jump (forward) and takes value 0, if particle stay at the same place, $T = \{0, h, 2h, \ldots\} (h > 0), t \in T$.

1) **Symmetric model.** Introduce the following expression for motion:

$$P\{\varepsilon_{i,t} = d \mid \rho_{i,t} = kd\} = r_k^i,$$

$$k = 2,\dots,N-s+1;\ r_k^i + l_k^i = 1$$

$$P\{\varepsilon_{i,t} = 0 \mid \rho_{i,t} = kd\} = l_k^i,$$

$$P\{\varepsilon_{i,t} = d \mid \rho_{i,t} = d,\ \varepsilon_{i+1,t} = d\} = r_1^i,$$

$$r_1^i + l_1^i = 1$$

$$P\{\varepsilon_{i,t} = 0 \mid \rho_{i,t} = d,\ \varepsilon_{i+1,t} = d\} = l_1^i,$$

$$P\{\varepsilon_{i,t} = d \mid \rho_{i,t} = d,\ \varepsilon_{i+1,t} = 0\} = 0,$$

$$P\{\varepsilon_{i,t} = 0 \mid \rho_{i,t} = d,\ \varepsilon_{i+1,t} = 0\} = 1,$$

Model with a leader. We'll assume that there is a leader particle (let's take the particle with the number 1), which has some advantage and moves with the following parameters

$$P\{\varepsilon_{1,t} = 1(0) \big| \rho_{1,t} = k\} = r(l) \quad ,r > 0,\ r + l = 1,\ k = 2 \dots n-1$$

other particles move as in symmetric model of motion. If the distance between leader and any other particle becomes equal d, then leader pushes this particle forward and that particle can push other if the distance between them also equal d.

Simulation. Let's take $N = 40$ and number of particles s takes values from 2 to 40. Consider two different models:

Model 1. In the model, the leader's parameters equal

$$r = 0.6;\quad l = 1 - r = 0.4.$$

Other particles move as in asymmetric model i.e. probability of jump for each particle depends on a distance to the next particle and have the same parameters:

$$r_k^i = 0.7;\ l_k^i = 1 - r_k^i = 0.3\ \ i = 2,\dots,s.$$

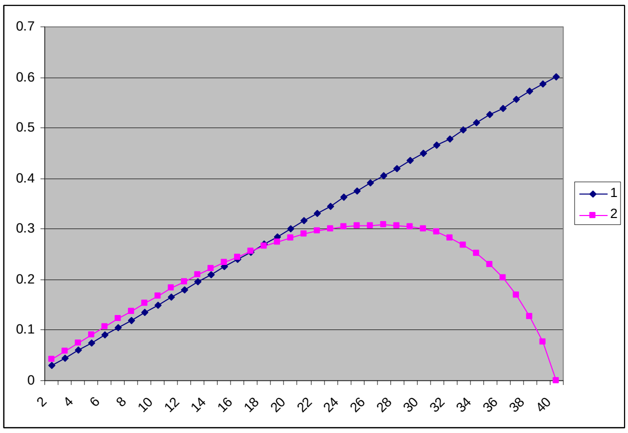

fig. 2

Model 2. In the symmetric model all particles move with the same parameters

$$r_k^i = 0.7\,;\; l_k^i = 1 - r_k^i = 0.3\,,$$
$$i = 1,2,...,s\,;\; k = 1, n - s + 1\,.$$

Simulation time is $T = 100000h$. As it is considered the stationary regime, hence we start since step $10001h$. Our goal is to estimate the traffic output capacity i.e. number of particles crossing a fixed point in a unit time. Results of the observation is given in the fig 2.

References

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