SECOND ORDER NECESSARY OPTIMALITY CONDITIONS AND INVESTIGATION OF SINGULAR CASES IN THE PROBLEM OF OPTIMAL CONTROL OF LAUDED DIFFERENTIAL EQUATIONS WITH MULTI-POINT NON-LOCAL BOUNDARY CONDITIONS

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As present, theory of higher order necessary optimality conditions, in particular, theory of singular controls has been completely worked out in the problems of optimal control of the systems with concentrated and distributed parameters under different local boundary conditions. The more detailed review of appropriate results is in the papers [1-3] and etc.

Recently, for different optimal control problems with non-local boundary conditions, necessary optimality conditions were obtained in the form Pontryagins maximum principle, of linearized maximum principle in the papers of O.O. Vasil'eva, O.O. Vasil'eva and K. Mizukami, F.Sh. Akhmedov, V.M. Abdullayev, Ya.A. Sharifov, K.B. Mansimov, F.Sh. Akhundov and K.M. Aliyev and others.

In several papers of O.V. Vasil'eva (see [4-8]), in the problem of control of ordinary dynamic systems with non-local boundary conditions of the form

$$A x(t_0) + B x(t_1) = \ell , \tag{1}$$

the analogy of Pontryagins maximum principle was proved and numerical methods were suggested. Pontryagin's maximum principle from the papers [4-8] was later repeated in the papers [9, 10].

The singular in the sense of Pontryagin's maximum principle case in control problem with non-local boundary conditions of the form (1) was studied in the papers [6, 7, 8] of O.V. Vasil'eva and K. Mizukami.

Therewith, R. Gabasov and F.M. Kirillova's [12] method of matrix impulses was used. Later, the similar result was obtained in the paper [11] in another form by means of the method suggested by K.M. Mansimov in [1,2] ana etc.

We can assume that the results of the papes [6] and [10] are equivalent.

In [12], F.Sh. Akhmedov has considered a general problem with non-local boundary conditions of the form

$$\sum_{i=1}^{k} B_i\, x(\xi_i) = \ell, \qquad (\xi_i \in [t_0, t_1]) \tag{2}$$

for loaded ordinary differential equations.

The analogy of Pontryagin's maximum principle was proved, the singular in the sense of Pontryagin's maximum principle case was investigated. Unfortunately, in more later papers of another authors there is no reference to this paper of the author.

In the present paper, we consider a problem on the minimum of the functional

$$S(u) = \varphi(x(\xi_1), x(\xi_2), \ldots, x(\xi_k)), \tag{3}$$

under the following constraints

$$\dot{x}(t) = f(t, x(t), x(\xi_1), x(\xi_2), \ldots, x(\xi_k), u(t)), \qquad t \in [t_0, t_1], \tag{4}$$

$$\sum_{i=1}^{k} B_i\, x(\xi_i) = \ell, \tag{5}$$

$$u(t) \in U \subset R^r, \qquad t \in [t_0, t_1]. \tag{6}$$

It is assumed that the data of problem (3)-(6) satisfy the smoothness conditions of type [12, 13] the are necessary for correctness of carried out arguments.

Assume that $(u(t), x(t))$ is a fixed admissible process.

Introduce the denotation

$$H(t, x, a_1, a_2, ..., a_k, u, \psi) = \psi' f(t, x, a_1, a_2, ..., a_k, u),$$

$$\frac{\partial f[t]}{\partial u} \equiv \frac{\partial f(t, x(t), x(\xi_1), ..., x(\xi_k), u(t))}{\partial u},$$

$$\frac{\partial H[t]}{\partial a_i} \equiv \frac{\partial H(t, x(t), x(\xi_1), ..., x(\xi_k), u(t), \psi(t))}{\partial a_i}.$$

Here $\psi(t)$ is the solution of the problem

$$\psi(t) = -\sum_{j=1}^{k}\left[\frac{\partial \varphi(x(\xi_1), ..., x(\xi_k))}{\partial a_j} e(\xi_j - t) - \int_{t_0}^{t_1} \frac{\partial H[t]}{\partial a_j} e(\xi_j - \tau) d\tau + \lambda_j B_j e(\xi_j - t)\right] +$$
$$+ \int_{t_0}^{t_1} \frac{\partial H[t]}{\partial x} e(t - \tau) d\tau,$$

where the constant vector λ is determined from the relation

$$\sum_{j=1}^{k}\left[\frac{\partial \varphi(x(\xi_1), ..., x(\xi_k))}{\partial a_j} - \int_{t_0}^{t_1} \frac{\partial H[t]}{\partial a_j} dt - \lambda' B_j\right] - \int_{t_0}^{t_1} \frac{\partial H[t]}{\partial x} dt = 0$$

Here, $e(t)$ is Heaviside's function.

Under some assumptions, it follows from the results of the papers [12, 13] that if U is open, then for the optimality of the admissible control $u(t)$ in the problem (3)-(6), it is necessary that for almost all $\theta \in [t_0, t_1]$ the relation

$$H_u[\theta] = 0 \quad \text{(Euler equation).} \tag{7}$$

be fulfilled.

Call each admissible control being a solution of Euler's equation a classic extremals.

Let the pair $F(t) = (F_0(t), F_1(\tau, t))$ be a solution of the problem [12].

$$\sum_{j=1}^{k} B_j F_0(t) - \int_{t_0}^{t_1}\left[\sum_{j=1}^{k} \frac{\partial f[s]}{\partial a_j} + \frac{\partial f[s]}{\partial x}\right] F_1(s, t) ds = E,$$

$$F_1(\tau, t) + \sum_{j=1}^{k} e(\xi_j - \tau) B_j F_0(t) - \int_{t_0}^{t_1}\left[\sum_{j=1}^{k} e(\xi_j - \tau)\frac{\partial f[s]}{\partial a_j} + e(s - \tau)\frac{\partial f[s]}{\partial x}\right] F_1(s, t) ds =$$
$$= e(t - \tau) E, \quad \tau \in T.$$

Following [1], introduce the matrix function in

$$M(\tau, s) = -\sum_{i,j=1}^{k} F_1'(\tau, \xi_i) \frac{\partial^2 \varphi(x(\xi_1), ..., x\xi_k)}{\partial a_i \partial a_j} F_1(s, \xi_j) -$$
$$- \int_{t_0}^{t_1}\left[F_1'(\tau, t)\frac{\partial^2 H[t]}{\partial x^2} F(s, t) + 2\sum_{i=1}^{k} F_1'(\tau, t)\frac{\partial^2 H[t]}{\partial x \partial a_i} F(s, \xi_i) + \sum_{i,j=1}^{k} F_1'(\tau, \xi_i)\frac{\partial^2 H[t]}{\partial a_i \partial a_j} F(s, \xi_j)\right] dt.$$

It is proved that for the optimality of the classic extremal, it is necessary that the inequality

$$\int_{t_0}^{t_1}\int_{t_0}^{t_1} \delta u'(\tau) \frac{\partial f[\tau]}{\partial u} M(\tau, s) \frac{\partial f[\tau]}{\partial u} \delta u(s) ds\, d\tau + \int_{t_0}^{t_1} \delta u'(\tau) \frac{\partial^2 H[t]}{\partial u^2} \delta u(t) dt + 2\sum_{i=1}^{k} \int_{t_0}^{t_1}\left[\int_{t_0}^{t_1} \delta u'(t) \frac{\partial^2 H[t]}{\partial u \partial a_i} \times\right.$$

$$\times F_1(\tau,\xi_i)\frac{\partial f[\tau]}{\partial u}\delta u(\tau)d\tau + 2\int_{t_0}^{t_1}\delta u'(t)\frac{\partial^2 H[t]}{\partial u\,\partial x}F_1(\tau,t)\frac{\partial f[\tau]}{\partial u}\delta u(\tau)d\tau \Bigg] dt \le 0\,, \qquad (8)$$

be fulfilled for all $\delta u(t) \in L_\infty(T, R^r)$.

Notice that inequality (8) is a second order integral necessary optimality condition of second order. Similar optimality conditions for different optimal control problems with local boundary conditions earlier were obtained by K.B. Mansimov [1,2,15-17]. The similar result was also obtained for quasisingular [1,2,15,16] controls.

We have investigated the singular in the sense of Pontryagin's maximum principle controls. Multi-point necessary optimality conditions of type [1,2,15] were obtained. These results were announced in [14]. In the full volume they will be published separately.

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