The Third International Conference "Problems of Cybernetics and Informatics"
September 6-8, 2010, Baku, Azerbaijan. Section #3 "Modeling and Identification"
www.pci2010.science.az/3/35.pdf

IDENTIFICATION OF NONLINEAR DYNAMIC SYSTEMS WITH POSITIVE FEEDBACK

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Systems, usually functioning with a positive feedback are widespread in chemical, mining, pulp-and-paper industry, ecologies, etc. Such systems are differed by the fact, that a part of the initial material remained unprocessed up to the required condition, when passing through to the working part of the object, returns to the entry of this object forming a closed or a so called recirculatory cycle. Systems with a closed cycle are characterized by the maximum raw material utilization and comparatively high efficiency [1, 2]. They are complex nonlinear control objects - the steady movement at their output is reached only at the certain values of the parameters of the system and under the change of the input influence within certain limits.

When constructing models of such systems the circumstance that there is certain a priori information about the system should be taken into consideration. For example there is certain information about the static characteristic of the system for the mill of ore-dressing plant working with a closed cycle, proceeding from their functioning conditions, which can be approximated by a polynomial function of the second degree. As to the system sluggishness, it is considered in the form of linear dynamic - in particular, aperiodic elements [2]. Therefore we can use block-oriented models with a feedback [3] for modelling systems with a positive feedback.

The problem of structural identification of nonlinear closed systems was considered earlier on the set of continuous block-oriented models with the feedback, consisting in Hammerstein and Wiener models with unit feedback [4, 5] and on the "greater" set of block-oriented models [6].

A considerable quantity of scientific works is devoted to the problem of parametric identification of nonlinear systems in which this problem is solved on the basis of different approaches and methods. At representation of nonlinear systems by the block-oriented models, most of the methods of the parametric identification is developed for the simple Hammerstein model (for example [7]). A comparatively small quantity of works is devoted to the identification of parameters of the simple Wiener model (for example [8]). As to the identification of parameters of other opened block-oriented models and block-oriented models with the feedback, success in this field is insignificant.

In the given work the problems of structural and parametric identification of the nonlinear systems with positive feedback on the set of block-oriented models with the feedback are considered. The problem of parametric identification can be connected directly to the problem of structural identification using the experimental data, received for solving the problem of structural identification.

The problem of structural identification, which is coordinated with Zadeh's classical definition of identification, is posed as follows: classes of models and input signals are given; it is required to develop a criterion identifying the model structures from the class of models.

In this work the method of parametric identification of nonlinear systems with the feedback, which allows to determine two types of parameters - static characteristics in the steady state, and dynamic characteristics in the transitive state on the basis of the least squares method is offered.

The structure of the model of nonlinear systems with the feedback is determined on the set of the continuous block-oriented models with feedback:

$$L = \{ s_i \,|\, i = 1,2,...,7 \}, \tag{1}$$

where s_1 is linear model with nonlinear feedback, s_2 is nonlinear model with linear feedback, s_3 and s_4 are Hammerstein and Wiener models with unit feedback, s_5 is Wiener- Hammerstein cascade model with unit feedback, s_6 and s_7 are Hammerstein and Wiener models with linear feedback (Fig. 1).

On the basis of the a priori information it is supposed that the nonlinear static element, which is a part of block-oriented models with feedback, is described by the polynomial function of the second degree:

$$f[x(t)] = c_1 x(t) + c_2 x^2(t), \tag{2}$$

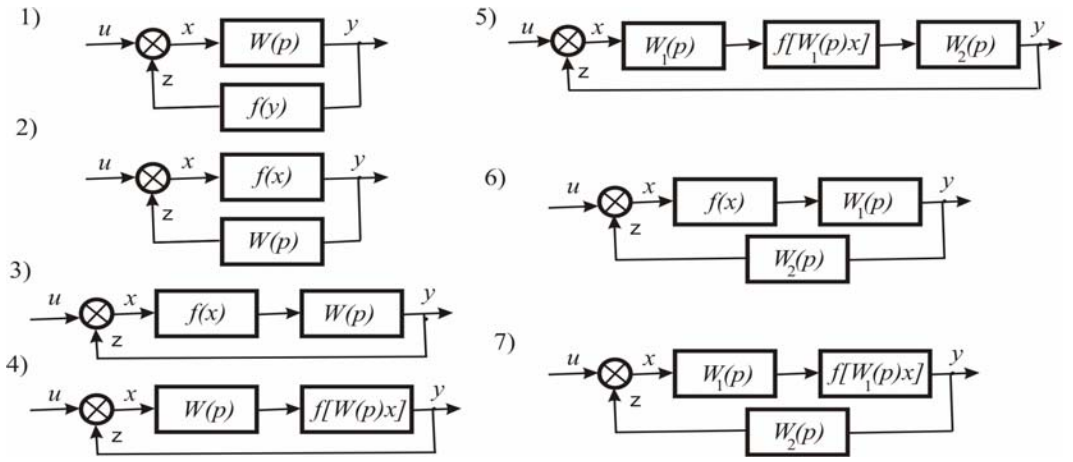
where the free member c_0 is absent, since there is no signal on the output of system with a feedback at the zero input signal.

The transfer functions of the linear dynamic elements have the following form:

$$W(p) = \frac{1}{Tp+1},\ W_i(p) = \frac{1}{T_i p + 1}\ (i = 1,2), \tag{3}$$

where T, T_i $(i = 1,2)$ - time constants, p designates the differentiation operation $p \equiv \dfrac{d}{dt}$.

Fig. 1. The block-oriented models with feedback: 1) linear model with nonlinear feedback; 2)



nonlinear model with linear feedback; 3) Hammerstein model with unit feedback; 4) Wiener model with unit feedback; 5) Wiener- Hammerstein cascade model with unit feedback;6) Hammerstein model with linear feedback; 7) Wiener model with linear feedback.

Due to (2) and (3) the models s_i $(i = 1,2,3,4)$ of the set of models (1) are described by the ordinary nonlinear differential equations of the first order - Riccati equation, and the models s_i $(i = 5,6,7)$ - by the ordinary nonlinear differential equations of the second order, which when $u(t)$ is harmonic function, are known in the theory of oscillations as Duffing equation.

For solving the problem of structural identification of nonlinear systems it is supposed that the input variable of the system $u(t)$ - a real periodic function with period T, for which exists uniformly and absolute convergent Fourier series. The class of such input signals includes such signals as a symmetric triangular impulse, symmetric trapezoidal impulse, half-sine impulse, etc.

In order to determine the identification criterion of the model structure on the set of the models (1) it is necessary to solve differential equations of the models, and also to consider stability conditions of the steady motion at the output of the nonlinear systems functioning with a positive feedback.

For a partial class of nonlinear systems with a feedback, so called recirculatory systems, it was obtained [2], that the implementation of the following conditions:

$$0 < c_1 < 1,\ \ 0 < c_2\,,\ \ \bar{u} < \frac{1-c_1}{2c_2}\,, \tag{4}$$

where $\bar{u}$ is a value of the input signal for some steady state, guarantees the system stability. Therefore for the models of the set (1) it is supposed, that the conditions (4) are valid.

Due to the conditions (4) in the models $s_i \ (i = 5,6,7)$ a nonresonance case takes place; besides according to the Diulak criterion [9], in these models there can not be self-oscillations. According to the above mentioned, for the solution of Riccati and Duffig equations, corresponding to the models, it is possible to use the method of a small parameter and to search for the solution of the equations in the form of the following series:

$$y(t) = \sum_{n=1}^{\infty} \mu^n y_n(t), \tag{5}$$

where μ is a small parameter.

Substituting (5) in the differential equations of models and comparing coefficients under identical degrees μ in the left and right parts of the equation, we get recurrent formulas for determining $y_n(t)\ (n = 0,1,\ldots)$. Using such formulas for determining $y_n(t)$ we get uniformly and absolutely convergent series. Hence we can use the Cauchy rule for the multiplication of such series and differentiation and integration operations can be done inside the sums.

As a result of solving the differential equations of the models of the set (1) are obtained expressions of the output variables in the steady state, which are not given here due to their big volume.

The Analysis of these expressions gives possibility to determine a choice criterion of the model structure on the set (1):

- The nonlinear model with the linear feedback - a constant component of the output periodic signal does not depend on the change of the period of input periodic signal;
- Hammerstein Models (with unit and linear feedback) - a constant component of the output periodic signal is decreased when increasing the period of the input periodic signal increases;
- The linear model with nonlinear feedback, Wiener models (with unit and linear feedback) and Wiener-Hammerstein cascade model with unit feedback - a constant component of the input periodic signal is increased when increasing the period of the input periodic signal.

Due to the functioning peculiarities of nonlinear systems with the feedback and mathematical difficulties the solution of the parametrical identification problem of such systems is much more complex than identification problem of open nonlinear systems.

The majority of block-oriented models with the feedback are nonlinear concerning parameters and the analytical solution of the parametrical identification problem is possible for some low order models.

For the models of the set (1) the connection between input and output variables in the steady state is determined by the equation

$$(1-c_1)x(t) - c_2 x^2(t) = u(t)\,,\quad x(t) = u(t) + y(t)\,. \tag{6}$$

Let's assume that u_i, $y_i\,(i = 1,2,\ldots,n)$ which are the values of system's input and output variables Estimate $u(t)$ and $y(t)$ are known in the steady state at the moments $t_i\ (i = 1,2,\ldots,n)$.

Estimates c_1 and c_2 coefficients are obtained based on the least squares method, when the sum

$$\sum_{i=1}^{n} \left[u_i - (1-c_1)x_i + c_2 x_i^2\right]^2 \tag{7}$$

gets a minimum value.

Differentiating (7) by c_1 and c_2, equating the obtained expression to zero, we receive the system of equations concerning these parameters. After solving this system of equations, we get:

$$c_1 = 1 - \frac{\left(\sum_{i=1}^{n} x_i^4\right)\left(\sum_{i=1}^{n} u_i x_i\right) + \left(\sum_{i=1}^{n} x_i^3\right)\left(\sum_{i=1}^{n} u_i x_i^2\right)}{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} x_i^4\right) - \left(\sum_{i=1}^{n} x_i^3\right)^2}, \quad c_2 = \frac{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} u_i x_i^2\right) + \left(\sum_{i=1}^{n} x_i^3\right)\left(\sum_{i=1}^{n} u_i x_i\right)}{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} x_i^4\right) - \left(\sum_{i=1}^{n} x_i^3\right)^2}. \quad (9)$$

Using the formulas (9) we can determine estimates of the parameters c_1 and c_2 for all models s_i $(i = 1,2,\ldots,7)$ of the set (1).

When estimating dynamic characteristics - time constants, it is assumed, that the estimations of static characteristics and the values of u_i, $y_i(i = 1,2,\ldots,n)$ in the transitive state are known. In this case the problem of the estimation of dynamic characteristics for each model of the set L is reduced to the problem of the solution of different kind algebraic equations relative to unknown parameters.

The reliability of the achieved results at the structural identification of nonlinear systems in the industrial conditions under noise and disturbances depends on the measurement accuracy of the system's output signals and on the mathematical processing of the experimental data.

The accuracy of the estimation of the dynamic characteristics in many respects is determined by the accuracy of definition of derivatives for input and output variables in the discrete time moments. Therefore, it is recommended to use the methods of numerical differentiation based on the A.N. Tikhonov regularization method for obtaining the acceptable results.

The developed methods and algorithms of structural and parametric identifications are investigated by means of both the theoretical analysis and the computer modelling based on using MATLAB.

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"The designated project has been fulfilled by financial support of the Georgia National Science Foundation (Grant # GNSF/ST08/3-392). Any Idea in this publication is possessed by the author and may not represent the opinion of the Georgia National Science Foundation itself".