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COORDINATE-WISE CONSIDERING SELF-RESTRICTION OF VARIABLES IN THE LORENZ MODEL

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The well-known Lorenz model is used in many sections of modern science.

$$\begin{cases} \dot{x} = -\sigma x + \sigma y \\ \dot{y} = rx - y - xz \\ \dot{z} = -bz + xy \end{cases} \tag{1}$$

At a certain interpretation of variables [1], this model also describes the economic processes in a large modern city.

It is well known that for some numerical values of r-parameter one can observe strange attractor - chaotic behavior of variables in the Lorentz model. Of course, in economic research of the metropolis functioning, strange attractor is inappropriate, highly unsatisfactory. That's why we propose to modify the Lorentz model, which is shown in considering self-restriction growth of each individual variable. This mentioned effect of self-restraint is also well known [2] in mathematical biophysics or biology is no less famous model of Volterra-Lotka. So, some modifications of the Lorentz model have the following form:

$$\begin{cases} \dot{x} = -\sigma x + \sigma y \\ \dot{y} = rx - y - xz - y^2 \\ \dot{z} = -bz + xy \end{cases} \tag{2}$$

$$\begin{cases} \dot{x} = -\sigma x + \sigma y \\ \dot{y} = rx - y - xz \\ \dot{z} = -bz + xy - z^2 \end{cases} \tag{3}$$

Fig. 1-4 demonstrate known in the literature results of numerical experiments for the Lorentz model with σ=10, b = 8 / 3 and r = 166 (strange attractor mode: Figure 1 - the behavior of integral curves Fig. 2-4 - phase portraits)

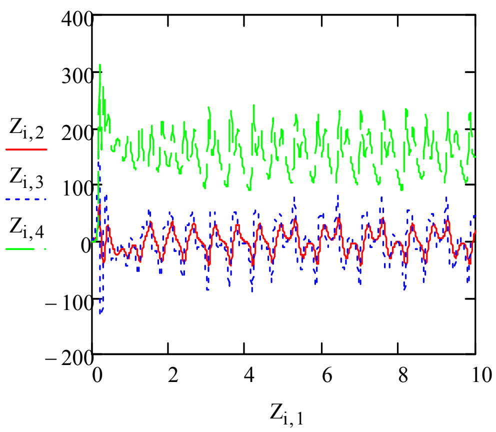

Fig. 1

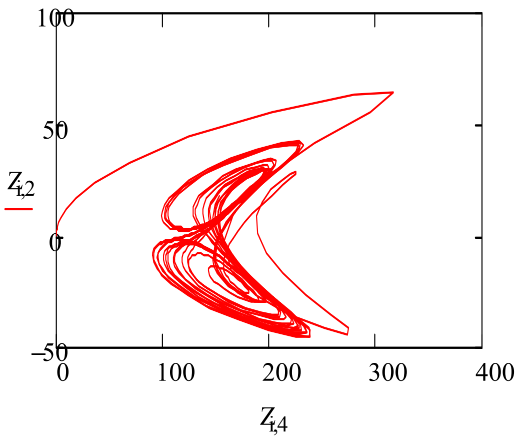

Fig. 2

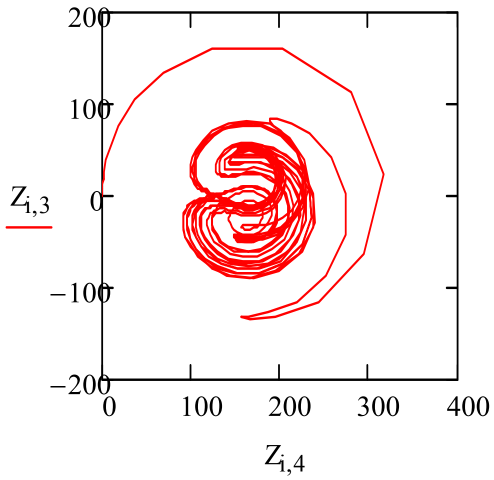

Fig. 3

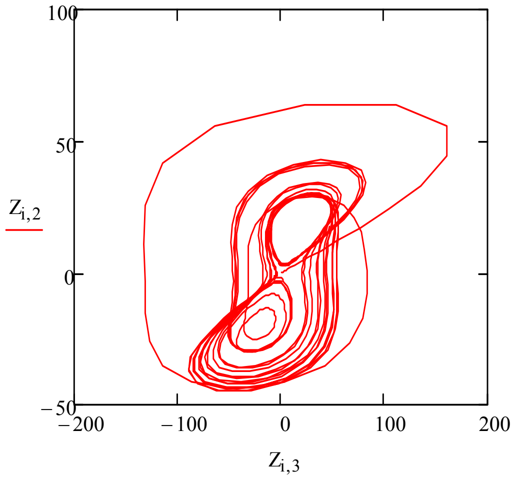

Fig. 4

where $Z_{i,1}$ is time, $Z_{i,2}$ is *x*, $Z_{i,3}$ is *y*, and $Z_{i,4}$ is *z*.

In the fig. 5-8 and 9-12, respectively we show the results of mathematical models calculation (2) and (3) with the same numerical values of the parameters of the Lorentz model.

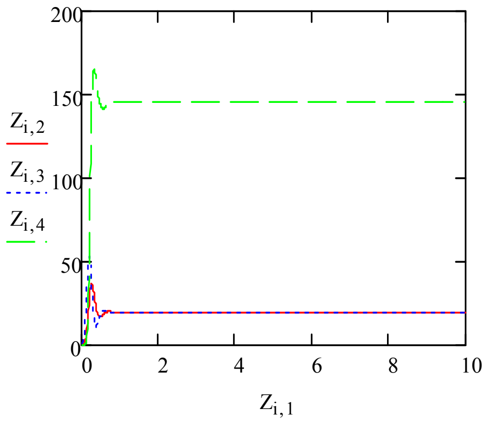

Fig. 5

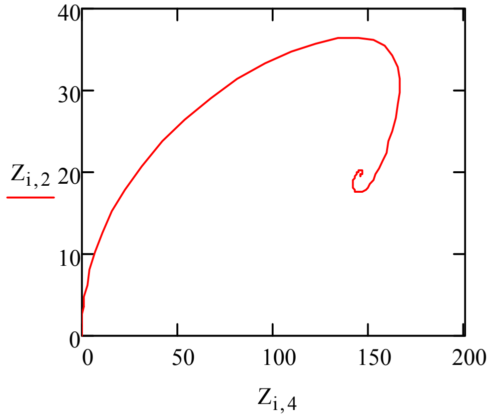

Fig. 6

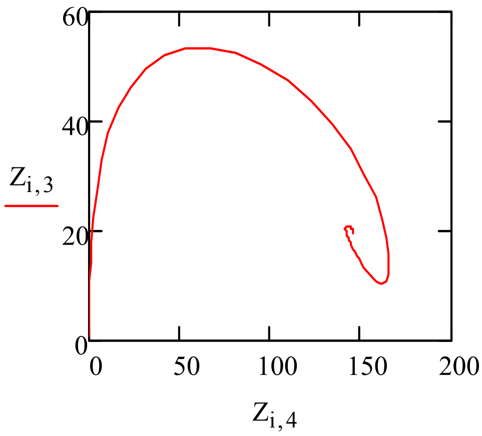

Fig. 7

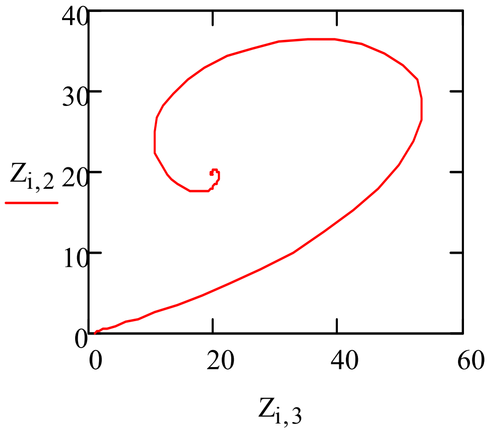

Fig. 8

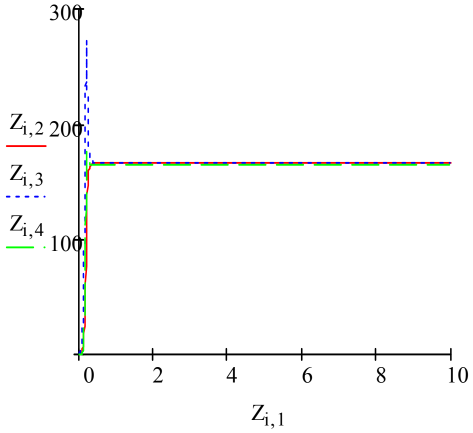

Fig. 9

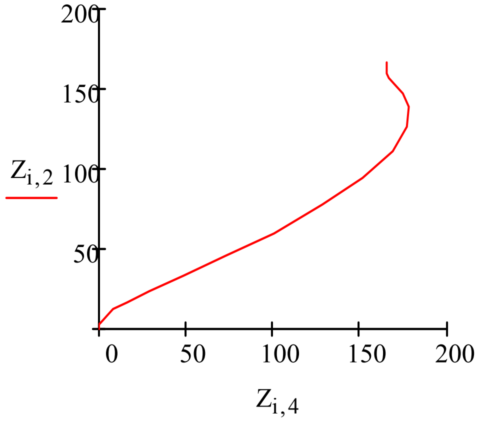

Fig. 10

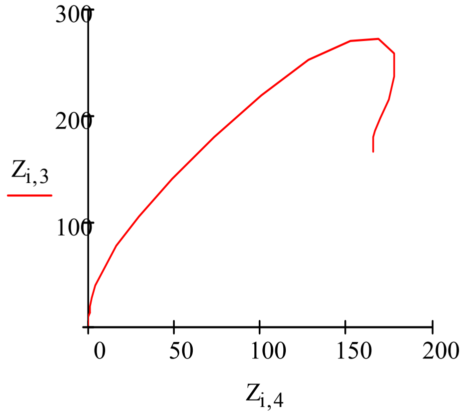

Fig. 11

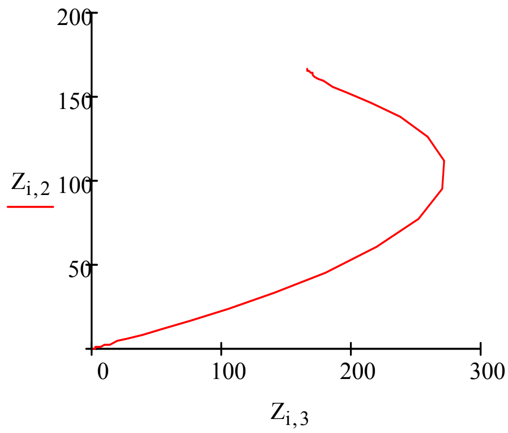

Fig. 12

As one can admit, there is no any strange attractor in both math models (2) and (3). The diversity of graphic simulation results obtained with the same initial conditions is especially interesting.

In conclusion, let us naturally raise some questions to be solved:

a) Is there any strange attractor in our math models (2) and (3)? Among the computational experiments carried out by us for r [0, 200] chaotic regime was not found.

б) What could be the nonlinearity in y^2 or z^2 that display self-restraint variable?

в) Will the relations $\frac{\dot{x}}{x}, \frac{\dot{y}}{y}, \frac{\dot{z}}{z}$ warn about the beginning of deterministic chaos?

References

1. Zang V.B. Synergetic Economics. Time and changes in nonlinear economic theory. - M.: Mir, 1999, 335 p.
2. Riznichenko G.Y., Rubin A.B. Biophysical dynamics of production processes. - Moscow-Izhevsk: Institute of Computer Science. - 2004, 464 p.
3. Grinchenko V.T. Matsipura V.T., Sparsky A.A. Introduction to nonlinear dynamics. Chaos and fractals. Publish. 2. - M.: Publishing LKI, 2007, 264 p.