ON CHOICE OF STATISTICAL ACCEPTANCE CONTROL BY THE ALTERNATIVE SIGN UNDER THE BAYESIAN APPROACH

Shakir Formanov[1], Anatoliy Startsev[2], and Tamara Formanova[3]

[1,2]Institute of Mathematics and Information Technologies, Tashkent, Uzbekistan
[3]Tashkent Automobile and Roads Institute, Tashkent, Uzbekistan
[1]*shakirformanov@yandex.ru*, [2]*anstartsev@gmail.com*, [3]*fortamara@mail.ru*

We consider the plan of statistical acceptance control (SAC) (n,k) consisting of the following: let there be a totality of N products subject to statistical acceptance control. A random sampling of the volume n is taken from this totality. If the amount of defect products in it $d_n \le k$, then the totality is accepted, otherwise it is rejected. The number k is said to be the acceptance number. The given version of quality control of ready product is called the plan of SAC by the alternative sign (any products are divided on suitable or defect ones) and denoted as (n,k).

Study of (n,k)-type plans of SAC is based on the theory of unbiased statistical estimates worked out by A.N. Kolmogorov [1] (see also [2], [3]). In the present work, the Bayesian approach is applied to problems of SAC. According to it, the sample volume n is considered as a fixed number and the part of defect products in the totality p is a random variable with the distribution function $G(p)$ (The monograph [4] by A. Hald is devoted to general mathematical concepts of application of the Bayesian approach).

Introduce the following cost parameters connected with conducting SAC: let a be a damage of a defect product in accepted party, b be a damage of a defect product in rejected party, c be a cost of checking of a product in the sample, l be a cost of checking of a product under total checking. Parameters a and b do not depend on the control method and for any special cases, $c \approx l$. The important agreement is decision about the rejected set. Throughout what follows we consider that the rejected set is checked completely and defect products are changed by suitable ones. The last makes clear the meaning of the parameter b. Such interpretation of economical parameters, connected with any plan of SAC, is received in [3], [5], [6].

If we denote by $W(p,n,k)$ the operative characteristics of the plan (n,k) and preserve notations for introduced cost parameters, then mean lose (see [5], [6])

$$L(p,n,k) = ApW(p,n,k) + (Bp+E)(1-W(p,n,k)) + nc$$

where

$$A = Na, \quad B = Nb, \quad E = Nl.$$

Hence, the Bayesian risk function

$$R(n,k) = \int_0^1 L(p,n,k)\,dG(p). \tag{1}$$

The value $k = k_0$ is called the optimal acceptance number if

$$R(n,k_0) = \min_{0\le k\le n} R(n,k). \tag{2}$$

If we assume that the population size is sufficiently large, then the distribution of d_n is subjected to the binomial law, i.e. the operative characteristic has the form

$$W\left(p,n,k\right)=\sum_{i=0}^{k}C_n^i p^i\left(1-p\right)^{n-i}. \qquad (3)$$

Set

$$m_k=\int_0^1 p^{k+1}(1-p)^{n-k}\,dG\left(p\right),\ \ l_k=\int_0^1 p^k\left(1-p\right)^{n-k}\,dG\left(p\right).$$

It is easy to see that the ratio $y_k=\dfrac{m_k}{l_k}\le 1$ does not decrease with growth of k.

With respect to the optimal acceptance number of (n,k) plan the following statements are taken place. In this connection we keep in mind the concepts (1)-(3).

Theorem 1. The optimal acceptance number is such value k_0 for that

$$Y_{k_0}\le p_0,\qquad Y_{k_0+1}\ge p_0,$$

where $p_0=\dfrac{l}{a-b}$ is the indifference part.

In particular, when

$$dG\left(p\right)=\frac{1}{B\left(\alpha,\beta\right)}p^{\alpha-1}\left(1-p\right)^{\beta-1}dp,$$

where $\alpha>0,\ \beta>0$ and $B\left(\cdot,\cdot\right)$ is the Euler $\beta-$function, then the optimal acceptance number is equal to

$$k_0=\left[\left(n+\alpha+\beta\right)p_0-\alpha\right]$$

Here $\left[x\right]$ is the integer part of number x.

It is known that in general case the number of defect product d_n has a hypergeometric distribution. If $np\to\lambda,\ \ \dfrac{n}{N}\to 0,$ then this distribution by a Poisson distribution is approximated.

Therefore, in this case we can suppose that the operative characteristic has the form

$$W\left(p,n,k\right)=\pi\left(\lambda\right)=\sum_{i=0}^{k}\frac{\lambda^i}{i!}e^{-\lambda}.$$

Let λ is random variable with the distribution function $F\left(x\right)$ and

$$\theta_k=\int_0^\infty x^k e^{-x}dF\left(x\right).$$

Theorem 2. If λ has a priori distribution $F\left(x\right),$ then the optimal acceptance number k_0 satisfy to the following inequalities

$$\frac{\theta_{k_0+1}}{\theta_{k_0}}\le np_0,\qquad \frac{\theta_{k_0+2}}{\theta_{k_0+1}}\ge np_0.$$

In particular, if λ has the exponential distribution with parameter $\lambda_0,$ i.e.

$$F(x) = \begin{cases} 0 & if \;\; x \le 0 \\ 1 - e^{-\lambda_0 x} & if \;\; x > 0 \end{cases}$$

then $k_0 = \left[np_0 \left(1 + x_0\right) - 1 \right]$.

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