ESTIMATION OF PARAMETERS OF CHI-SQUARE STATISTICAL STRUCTURES

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Let there is given (E,S) measurable space and on this space there is given μ_a, $a \in A$ family of probability measures depended on $a \in A$ parameter. Let bring some definitions (see [1]--[2]).

Definition 1. The following object {E,S, μ_a, $a \in A$} called chi-square statistical structure connected with a stochastic system, if $\forall a, a \in A$ μ_a is chi-square measures on S.

Definition 2. A statistical structure {E,S, μ_a, $a \in A$} connected with a stochastic system is called orthogonal (singular) if $(\forall i)\ (\forall j)\ (i \in A,\ j \in A, i \neq j \Rightarrow \mu_i \perp \mu_j)$

Definition 3. A statistical structure {E,S, μ_a, $a \in A$} connected with stochastic system is said to be weakly separable, if there exists a family of S-measurable sets $X_a, a \in A$ such that the relations $(\forall i)\ (\forall j)\ (i \in A,\ j \in A, i \neq j \Rightarrow \mu_i(X_j) = \begin{Bmatrix} 0, if\ i \neq j, \\ 1, if\ i = j \end{Bmatrix}$

Definition 4. A statistical structure {E,S, μ_a, $a \in A$} connected with a stochastic system is said to be strongly separable, if there exist pairwise disjoint S-measurable sets $X_a, a \in A$ such that the relation $(i \in A,\ \mu_i(X_i) = 1, \forall i \in A$ is fulfilled.

Definition 5. A statistical structure {E,S, μ_a, $a \in A$} connected with a stochastic system will be said to admit a consistent estimate of parameters if there exists a measurable map g of the space (E,S) in $(A, \Im)$ such that $\mu_a\{x : g(x) = a\} = 1, \quad \forall a \in A$.

Theorem1. On a arbitrary set E of continuum power one can define chi-square ortogonal statistical structure connected with a stochastic system having maximal possible power equal 2^{2^c}, where c is continuum power.

Theorem2. On a arbitrary set E of continuum power one can define chi-square weakly statistical structure connected with a stochastic system having maximal possible power equal 2^c, where c is continuum power.

Theorem3. On a arbitrary set E of continuum power one can define chi-square strongly statistical structure connected with a stochastic system having maximal possible power equal c, where c is continuum power.

Theorem4. Let $M_H = \oplus H_2(\mu_a)$ be the Hilbert space of measures and E be a complete separable metric space and S be the Borel σ-algebra and $card(A) \le 2^{\chi_0}$. Then in the theory (ZFC)$(MA) for the chi-square ortogonal statistical structure {E,S, μ_a , $a \in A$ }to admit a consistent estimate of parameters $a \in A,$ it is necessary and sufficient that it admit on unbiased estimate of any parametric function g and the correspondence $f \to \psi_f$ given by the equality

$$\int f(x)\nu(dx) = \langle \psi_f, \nu \rangle, \forall \nu \in M_H$$

be one-to-one ($f \in F$, where F is the set of those f for which $\int f(x)\nu(dx)$ is defined $\forall \nu \in M_H$).

References

1. Z. S. Zerakidze, On consistent estimators for families ofprobability measures. 5-th Japan-- USSR Symposium on Probability Theory, Kyoto, 1986, 62--63.
2. Z. S. Zerakidze, Structure of the family of probability measures.Trudy Akad. Nauk Georgian SSR, 113 (1984), 37--39.