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ASYMPTOTIC BEHAVIORS OF THE CRITICAL BRANCHING PROCESSES WITH DECREASING IMMIGRATION

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Let μ_n be a number of particles of the Galton-Watson (G-W) branching process at the moment n $(n=0,1,\ldots,\ \mu_0=1)$ with the generating function (g.f.)

$$F(x)=\sum_{j=0}^{\infty} p_j x^j, \qquad p_j = P\{\mu_1 = j\}, \quad j=0,1,\ldots, \quad |x|\le 1.$$

If $\mu_n=0$, then, at the moment n, ξ_n particles immigrate to the population, where the number of particles evolves by the law G-W process with g.f. $F(x)$. The asymptotic behavior of branching processes with state-dependent immigration were studied by many authors (see [1-5]). Assume that the intensity of the immigration decreases tending to 0, when the number of descendent increases. Limit theorems for such processes have been studied in [6-8].

Let Z_n be a number of particles of this process at the moment n.

Suppose, that

$$F(x)=x+(1-x)^{1+\nu}L(1-x),$$

where $0<\nu\le 1$ and $L(x)$ is a slowly varying function (s.v.f.) as $x\to 0$.

Put

$$\alpha_n = E\xi_n, \qquad \beta_n = D\xi_n + \alpha_n^2 - \alpha_n.$$

Introduce the function

$$M(n)=\sum_{k=1}^{n}\frac{N(k)}{k^{1/\nu}},$$

where N(x) is a s.v.f. as $x\to\infty$ such that

$$\nu N^{\nu}(x)L(N(x)/x^{1/\nu})\to 1 .$$

We suppose that

$$\mathop{Sup}_{0\le k<\infty}\alpha_k<\infty, \qquad \mathop{Sup}_{0\le k<\infty}\beta_k<\infty, \ ,$$

$$0<\alpha_n\to 0, \qquad \beta_n\to 0, \qquad n\to\infty \ .$$

Denote

$$Q_1(n)=\alpha_n\sum_{k=0}^{n}(1-F_k(0)), \qquad Q_2(n)=(1-F_n(0))\sum_{k=0}^{n}\alpha_k,$$

where $F_0(x)=x, \quad F_{n+1}(x)=F\big(F_n(x)\big).$

We consider the case $\nu=1, \quad M(n)\to\infty$ as $\quad n\to\infty$.

Theorem. Assume that

$$\alpha_n\sim\frac{l(n)}{n^r},\ \beta_n=o\big(Q_1(n)\big), \quad n\to\infty,$$

where $0\le r\le 1$ and $l(n)$ is a s.v.f. as $\quad n\to\infty$. Then the following statements take place:

a) if $r = 0$, $Q_1(n) \to \theta$ as $n \to \infty$ and $0 < \theta < 1$, then

$$\lim_{n\to\infty} P\{Z_n > 0\} = \frac{\theta}{1+\theta},$$

$$EZ_n \asymp \frac{n}{M(n)}, \quad n \to \infty.$$

b) if $0 < r < 1$ or $r = 0$, $Q_1(n) \to 0$, $n \to \infty$, then

$$P\{Z_n > 0\} \sim Q_1(n),$$

$$EZ_n \asymp \frac{n\alpha_n}{1-r}, \quad n \to \infty.$$

c) if $r = 1$ and $\beta_n = o\left(Q_1(n) + Q_2(n)\right)$ as $n \to \infty$, then

$$P\{Z_n > 0\} \sim Q_1(n) + Q_2(n),$$

$$EZ_n \asymp M(n), \quad n \to \infty.$$

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