The Second International Conference "Problems of Cybernetics and Informatics"
September 10-12, 2008, Baku, Azerbaijan. Section #3 "Modeling and Identification"
www.pci2008.science.az/3/41.pdf

DEA: A METHOD FOR MEASURING THE EFFICIENCY OF AGENCIES

Rouzbeh Rahmanian

Islamic Azad University Rasht Barnch, Rasht, Iran, *Rouzbeh52@yahoo.com*

Man's economic attempts have always been made in a way that maximum results would be obtained with existing facilities and factors. This tendency can be called "Access to Higher Efficiency and Productivity." As a whole, productivity is a comprehensive concept encompassing the efficiency whose increase in order to improve the life quality, welfare, peace and well-being of people has always been considered by those in charge of politics and economy. The significance of the issue is to the point that some attribute the durability and consistency of a political-economic system to productivity. While mentioning his philosophical stand points regarding the issue, Lenin concluded his discussion with this sentence, "In a final analysis, work productivity is the most important subject and the most basic issue for the victory of our social system." Generally, the significance and recognition of productivity is not limited to capitalist and socialist schools and societies because it has been focused on in Islam and Islamic societies as well. In other words, productivity has a history of more than 1400 years yet its scientific evidences and practical measures have been suggested only in recent years [1].

Efficiency is the expression of the concept that how well an organization uses its resources in accordance with the best performance in a specified period of time to have a satisfactory production [8]. Generally, it is the criterion of an organization's performance which is based on the level of resources (inputs). In other words, efficiency is considered to be the level of resource consumption for the purpose of producing a certain amount of product. Usually, efficiency is calculated by the equation of output/input ratio. For instance, the efficiency of an industrial unit for which one input such as cost and one output such as income is determined, would be as follows: *Efficiency of an industrial unit* = *Output*/*Input*.

Comparing the efficiency of this unit with that of other similar industrial units, only a unit is efficient which has less cost (less input) in comparison with its constant and similar income (constant output) or shows more income (more output) with equal costs (equal input). If the denominator has only one input type such as workforce or capital, it would be called "Partial Productivity" for which work or capital productivity can be given as an example [4].

Usually, various methods are applied for measuring the technical efficiency of units, but as a whole there are two main methods for this cause which are Parametric and Non-parametric methods. In Parametric Method a certain production function is estimated based on different statistical and econometrics methods. Then, this function is used in determining the efficiency level. In Non-parametric Method there is no need for estimating the production function. Data Envelopment Analysis (DEA) is a non-parametric method which evaluates the relative efficiency of units in comparison with each other. Here, no recognition of the production function form is needed and there is no limitation of the number of inputs and outputs [2].

Measuring the efficiency level, due to its importance in evaluating the performance of a company or organization has always been paid attention to by the researchers. In 1975, Farrell used a method quite similar to that of efficiency measurement in engineering issues to measure the efficiency of a production unit. The case he took into account for measuring efficiency included an input and an output and used this model to estimate the efficiency of the US agriculture section in comparison with other countries. However, he was not successful in presenting a method which could encompass various inputs and outputs [2].

Therefore, Charnes, Cooper and Rhodes developed Farrell's approach and presented a model that could measure efficiency with several inputs and outputs. This model was called "Data Envelopment Analysis" and for the first time, upon Cooper's consultation, Edward Rhodes used it in his dissertation entitled "Evaluation of the Educational Progress of the Students of the Schools in the United States of America" in 1976 in Carnegie University. In 1978, it was introduced in an article entitled "Measuring the Efficiency of the Decision Making

Units." Since this model was presented by Charnes, Cooper and Rhodes, it is known as the CCR model which is taken from the first letter of their names [6].

Data Envelopment Analysis (DEA) is a technique in which all collected observations of efficiency measurement are used. Unlike the regression method that by obtaining the mean value during the comparison of the units gives the best existing performance in the set of studied units, DEA optimizes each one of the observations in comparison with the efficient interface. Generally, in both parametric and non-parametric methods (Mathematical Planning) all information is used as a whole. In the parametric method, the performance of each unit against an optimized regression equation is determined while in DEA, the performance of *n* studied units is evaluated by making and solving *n* models. The figure below shows the comparison between these two techniques.

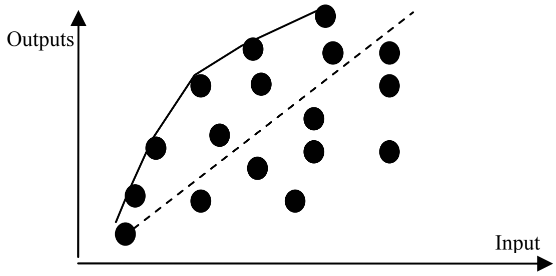

Fig. 1 Comparison of DEA and Regression [3]

The non-parametric method needs a mathematical function based on which by the application of independent variables, the dependent variable is estimated. Moreover, hypotheses regarding data distribution function along with the model's limitations should be taken into account. However, Data Envelopment Analysis does not require obtaining a distribution function and formulating its related hypotheses. In all, by combining all the studied units, this method compares a virtual unit having the highest efficiency with inefficient units [2]. Some advantages of the DEA technique in comparison with parametric methods like regression are as follows [7].

As mentioned before, this method makes measuring the units' relative efficiency possible. By measuring relative efficiency, it is meant comparing the efficiency of one unit with others that have relatively similar inputs and outputs. There are various methods for this measurement some of which are commonly suggested methods in economics. As a whole, there are two main concepts for determining units' efficiency by using DEA that are mentioned below:

- Usually, DEA is based on the hypothesis that if unit A can give more outputs in comparison with unit B, of course with the same amount of input (similar and equal inputs), the former unit would be considered more efficient than the latter.
- If unit A with a specified level of input gives a certain output, it can be expected that other similar units could have the same output with the same input and likewise, if unit B with a certain input is capable of producing a certain level of output, then again it can be expected that other units would have this capability as well. Now, it is possible to combine units A and B with other units and make a combination consisting of the units' inputs and outputs. However, since there is no unit with characteristics of such a combination, a virtual unit is made.

Generally, finding "the best" virtual unit by combining all real units is the heart of DEA. Now, if this unit is better than the studied unit, i.e. with the similar and equal inputs of the studied unit, it would give more outputs. Or if for similar and equal outputs it needs fewer inputs, the studied unit could be regarded as inefficient [2]. As a whole, DEA is a multifactor analytical model of productivity for measuring the relative efficiency of the Decision-Making Units (DMUs). In this model, the efficiency rate is defined with several inputs and outputs:

$$\text{Efficiency} = \frac{\text{Balance sum of outputs}}{\text{Balance sum of inputs}}$$

If the objective is to study the efficiency of *n* DMUs each of which having *m* inputs and *s* outputs, the relative efficiency of every unit for instance, unit zero, would be obtained by the following model [5]: Max Z = (The efficiency of the unit zero); St: Efficiency of all units ≤ 1. Its mathematical model (the CCR model) is as follows:

$$Z = \frac{\sum_{r=1}^{s} U_r Y_r}{\sum_{i=1}^{m} V_i X_i} \to \max\,;\ \frac{\sum_{r=1}^{s} U_r Y_r}{\sum_{i=1}^{m} V_i X_i} \le 1\,;\ V_i, X_i \ge 0, \tag{1}$$

where, X_{ij} is the i^{th} input's value for the j^{th} unit (*j*= 1, 2, 3,…,*m*); Y_{rj} is the r^{th} output's value for the j^{th} unit (*r*= 1, 2, 3,…,*s*); U_r is the weight given to the r^{th} output (the price of the r^{th} output); V_i is the weight given to the i^{th} input (the cost of the i^{th} input).

In the above-mentioned case, if U_r and V_i values are very large and very small respectively, the expressive ratios of the limitations would be infinite and unlimited. To prevent such problems, all ratios (efficiency of the units) should be considered smaller or equal to 1 and enter the model as a limitation.

The efficiency can be studied from two approaches of focusing on inputs and outputs. Charnes, Cooper and Rhodes (1981) defined efficiency with consideration of two approaches (Input/Output-centered): 1) In the input-centered model, a unit is regarded inefficient when reducing each one of the inputs without increasing others or when reducing each output is possible. 2) In the output-centered model, a unit is inefficient only when the possibility of increasing each one of the outputs without increasing an input or reducing another output exists.

Therefore, a unit is efficient if and only if none of the two aforesaid cases is possible. In addition, efficient level less than 1 for a unit means that the linear combination of other units can produce the same amount of output by using less input [2].

Input-centered CCR Multiple Model. To change the CCR ratio model into a linear planning model, Charnes and Cooper argue that to maximize the value of a fractional phrase the denominator has to be assumed a *constant* and the numerator, maximized. Thus, the denominator is 1 and so a new model will be obtained which is called the *Multiple Model*.

$$Z = \sum_{r=1}^{s} U_r Y_r \to \max\,;\ \sum_{i=1}^{m} V_i X_i = 1\,;\ \sum_{r=1}^{s} U_r Y_r - \sum_{j=1}^{n} X_{ij} V_i \le 0\,;\ U_r, V_i \ge 0\,. \tag{2}$$

Input-centered CCR Envelopment Model. Charnes, Cooper and Rhodes in making the Data Envelopment Analysis model came to an experimental equation in relation to the number of evaluated units and inputs/outputs which is as follows:

Number of evaluated units $\ge$ 3 (*number of inputs* + *number of outputs*).

Not using this equation would practically result in many units being put on the efficient interface and so, having an efficient value of 1. Therefore, the model's separation power will decrease. Since for each unit a certain limitation is written that leads to a linear planning model the number of limitations of which is more than the number of its variables and because the scope of operations in the simplex solution is more dependent on the number of limitations than the variables, then solving the secondary problem of the model above would require fewer operations. In case a corresponding variable with $\sum_{i=1}^{m} V_i X_{ij} = 1$ limitation is expressed in the secondary problem by θ or corresponding variables with $\sum_{l=1}^{m} Y_{rj} U_r - \sum_{l=1}^{n} X_{lj} V_l \le 0$ limitations are expressed by λ_j, the CCR secondary model would be as follows:

$$Y = \theta \to \min\,;\ \sum_{j=1}^{n} \lambda_j Y_{rj} \ge Y_r\ (r = \overline{1,s})\,;$$
$$\theta X_i - \sum_{j=1}^{n} \lambda_j X_{ij} \ge 0\,(i = \overline{1,m})\,;\ \lambda_i \ge 0\,(j = \overline{1,n})\,. \tag{3}$$

Note that in the primary model, *m* inputs, *s* outputs and *n* units existed based on which the first problem had *n*+1 variables whose limitation was less than the first problem and its solution required fewer operations. The objective of this model is to reduce the input level against θ [2].

Output-centered Models. The previously explained models usually focused on inputs. Hence, they could determine the weights in a way that the relative efficiency of the studied unit (unit zero) in comparison with other units would reach the maximum level and therefore, no other unit could reach that output level with fewer inputs. As a whole, output-centered CCR multiple (primary) and Envelopment (secondary) models are as follows:

$$Z = \sum_{i=1}^{m} V_i X_i \to \min\,;\ \sum_{r=1}^{s} U_r Y_r = 1\,;$$

$$\sum_{r=1}^{s} U_r Y_{rj} - \sum_{j=1}^{n} X_{ij} V_i \le 0; (j = \overline{2,n})\,;\ U_r, V_i \ge 0\,. \tag{4}$$

In case we assume the variable correspondent to the first limitation of the model mentioned above by θ in the secondary problem and λ_j as the variable correspondent to other limitations of the primary model, the envelopment model should be as follows:

$$Y = \theta \to \max\,;\ \sum_{j=1}^{n} \lambda_j Y_{rj} \ge \theta Y_r\ (r = \overline{1,s});$$

$$\sum_{j=1}^{n} \lambda_j X_{ij} \le X_i\ (i = \overline{1,m})\,;\ \lambda_i \ge 0\,(j = \overline{1,n})\,. \tag{5}$$

The objective of the said model is to obtain the highest level of outputs [2].

Data Envelopment Analysis (DEA) is a new method for "analyzing operations" which is used for evaluating relative efficiency. Usually, linear planning (LP) is the basis of the method and since it is an optimization method, it is considered preferable over other efficiency analysis techniques. With regards to the characteristics of DEA and its being capable of presenting a comprehensive and unique measuring method for every unit of several inputs/outputs, it seems suitable for calculating the efficiency of economic agencies. Also, the possibility of assuming virtual variables for this model and using shadow prices helps its application in evaluating the efficiency of non-profit-making and governmental sections' services. The said method enjoys an ever-increasing development and has various applications in a way that until 2001 about 18,000 pages of literature on DEA were published among which 50 books are included.

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