GENERALIZED MATHEMATICAL MODEL OF SENSITIVE ELEMENTS OF VIBRATION DENSITY MEASURES OF LIQUID

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Abstract

This article is dedicated to the question of working out of generalized mathematical model of sensitive elements of vibration measures of density of liquid. For this purpose we put and solve a task about own bend vibrations of the uniform pipe, filled with a liquid and fastened at the end's in closings, which are elastic, concerning both linear and angular movements.

INTRODUCTION

Presently the vibration ones are most spread among the existing variety of current measures of density. Sensitive elements of these instruments are tubes, resounding at their own frequency; these tubes are filling with analysis liquid. In all well-known works [1-3] dedicated to working out and exploration of mathematical model of sensitive element of the vibration measure of density, the latter is considered in a form of a uniform pipe of the constant section with stiggey closed ends. But, in reality ends of the pipe are able to make linear and angular movements because of last toughness of the closings. In this connection working out and exploration of mathematical model of sensitive element of the vibration measure of density taking into account real conditions of closing of the ends is considered to be very actual task.

RESEARCH

The sensitive element of the vibration measure of density is designed in a form of uniform pipe, filled with analyses liquid. This is a pipe of constant section, its ends are fixed at the bearings, which are elastic with regard to both linear and angular movements. For we wonder about resonance vibrations of the pipe, the equation of its movement can be represented in a following form

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0\,, \qquad (1)$$

where E – is a model of elasticity of material of the pipe; J – is a moment of inertia of the diametrical section of the pipe; y – is a diametrical movement of the point of the resonators with x coordinate in a moment of period t.

We'll look for a private answer in a form of the product of two functions

$$y = U(x)T(\tau) \qquad (2)$$

When the equation (1) will be as:

$$EJU^{IV}T + mU\ddot{T} = 0$$

or, otherwise

$$\frac{EJU^{IV}}{mU} = -\frac{\ddot{T}}{T} = P^2 \quad ,$$

Where P – is still unknown constant. Thus, in shade of the equation (1) we get two following equities:

$$\ddot{T} + P^2 T = 0 \tag{3}$$

$$U^{IV} - \nu^4 U = 0, \qquad \left(\nu^4 = \frac{mP^2}{EJ} \right) \tag{4}$$

from answer of the equity (3)

$$T = A\sin(PT + \gamma)$$

Is clear that the P – constant represents its own frequency. It's necessary to apply to the answer of the equity (4) for determination of the a forenamed frequency:

$$U = AS(\nu x) + BT(\nu x) + CU(\nu x) + DV(\nu x) \tag{6}$$

Where A,B,C and D – are arbiter constants found from bordering conditions, and S(νx), T(νx), U(νx) and V (νx) are functions of A.N.Krilov [4], they are determined in a following way:

$$S(\nu x) = \frac{1}{2}(ch\,\nu x + \cos\nu x)$$

$$T(\nu x) = \frac{1}{2}(sh\,\nu x + \sin\nu x)$$

$$U(\nu x) = \frac{1}{2}(ch\,\nu x - \cos\nu x)$$

$$V(\nu x) = \frac{1}{2}(sh\,\nu x - \sin\nu x)$$

Bordering conditions are determined by the conditions of fastening of ends of the pipe, they will be written so:
When x=0

$$\begin{aligned} k_1 y &= -EI\,\frac{d^3 y}{dx^3}; \\ \mu_1 \frac{dy}{dx} &= EI\,\frac{d^2 y}{dx^2} \end{aligned} \tag{7}$$

When $x = l$

$$k_2 y = EI\,\frac{d^3 y}{dx^3};$$

$$\mu_2 \frac{dy}{dx} = EI\,\frac{d^2 y}{dx^2}$$

Here $k_i, \mu_i; i = 1,2$ – are accordingly linear and angular toughness of the bearings. It comes out from conditions at the left end (x=0) that:

$$\begin{aligned} A &= -\frac{EI\nu^3}{k_1} D; \\ B &= \frac{EI\nu}{\mu_1} C \end{aligned} \tag{8}$$

Conditions at $x = l$ lead to the following equations

$$k_2(AS(\nu l)+BT(\nu l)+CU(\nu l)+D(\nu l)=-EI\nu^3(AT(\nu l)+BU(\nu l)+CV(\nu l)+DS(\nu l);$$
$$\mu_2\nu(AV(\nu l)+BS(\nu l)+CT(\nu l)+DU(\nu l))=EI\nu^2(AU(\nu l)+BV(\nu l)+CS(\nu l)+DT(\nu l)) \qquad (9)$$

Taking into account (8) in (9) we get

$$\begin{aligned}
&D\left[-\frac{(EI)^2\nu^6T(\nu l)}{k_1}+k_2V(\nu l)+EI\nu^3S(\nu l)-EI\nu^3S(\nu l)\frac{k_2}{k_1}\right]+\\
&+C\left[EI\nu T(\nu l)\frac{k_2}{\mu_1}+U(\nu l)k_2+\frac{(EI)^2\nu^4}{\mu_1}U(\nu l)+EI\nu^3V(\nu l)\right];\\
&D\left[\frac{(EI)^2\nu^5U(\nu l)}{k_1}-\frac{EI\nu^4\mu_2V(\nu l)}{k_1}+\mu_2\nu U(\nu l)-EI\nu^2T(\nu l)\right]+\\
&+C\left[\mu_2\nu T(\nu l)-\frac{(EI)^2\nu^{\#}V(\nu l)}{\mu_1}+\frac{EI\nu^2S(\nu l)\mu_2}{\mu}-EI\nu^2S(\nu l)\right]
\end{aligned} \qquad (10)$$

Equating the determinate of the system (10) to zero, we get the following equity of frequency:

$$\begin{aligned}
&\left[-\frac{(EJ)^2\nu^6T(\nu l)}{k_1}+k_2V(\nu l)+EJ\nu^3S(\nu l)-EJ\nu^3S(\nu l)\frac{k_2}{k_1}\right]*\\
&*\left[\mu_2\nu T(\nu l)-\frac{(EJ)^2\nu^3V(\nu l)}{\mu_1}+\frac{EJ\nu^2S(\nu l)\mu_2}{\mu_1}-EJ\nu^2S(\nu l)\right]-\\
&-\left[EJ\nu T(\nu l)\frac{k_2}{\mu_1}+U(\nu l)k_2+\frac{(EJ)^2\nu^4U(\nu l)}{\mu_1}+EJ\nu^3V(\nu l)\right]*\\
&*\left[\frac{(EJ)^2\nu^5U(\nu l)}{k_1}-\frac{EJ\nu^4V(\nu l)\mu_2}{k_1}-\mu_2U(\nu l)\nu-EJ\nu^2T(\nu l)\right]=0
\end{aligned} \qquad (11)$$

Which can be solved only by the numerical method?

Let's examine the private case $k_1=k_2=\mu_1=\mu=\infty$ (tough closing up of both end's of the pipe), as applied to the equity (11). By the way of opening (11) and dividing to k_2 m_2 we get

$$U(\nu l)^2-T(\nu l)=0$$

or

$$ch\,\nu l\cos\nu l-1=0$$

It is clear, that the equity (12) represents equity of frequency of the pipe with toughly closed end. This shows a good coordination of the received result with a theory.

Let's take the received results to a form, which is convenient for the practice changing the circular frequency *P* in (4) with a constant of periodical process *f*, and also heeding meanings of *j* and *m*, we get

$$f=f_0\frac{1}{\sqrt{1+A\rho_L}}$$

where *f* is frequency of the own vibrations of the pipe, filled with a liquid of ρ_L density; $f_0 = \frac{\lambda_k}{8\pi d^2}\sqrt{\frac{E}{\rho_p}(n^2+1)}$ is a frequency of own movements of the empty pipe; λ_k is a constant, depending on conditions of fastening ends of the pipe and numbers of form of vibrations k; A= $\rho_p(n^2+1)$ is a constant of the pipe; ρ_p is a density of material of the pipe; $n = \frac{D}{d}$ is a relative thickness of walls of the pipe; *D* and *d* are external and internal diameters of the pipe.

The constant λ_k is found as a root of the equity (11).

CONCLUSION

The elaborated and generalized model of sensitive element of the vibration measure of density of liquid comprehends all possible at the practice cases of fastening of their end's Application of this model allows to evaluate influence of parameters of the closings at frequency of own vibrations of the sensitive element of the measure of density, and also to make a theoretical graduated characteristic of the apparatus.

References

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