GRAVITATING SPHALERON-ANTISPHALERON SYSTEMS

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The given talk is result of joint scientific researches led by Prof. Dr. Jutta Kunz from University Oldenburg, thanking support by the Volkswagenstiftung [1-5]. We present new classical solutions of Einstein-Yang-Mills-Higgs (EYMH) theory, representing gravitating sphaleron-antisphaleron pair, chain and vortex ring solutions. In these static axially symmetric solutions, the Higgs field vanishes on isolated points on the symmetry axis, or on rings centered around the symmetry axis. We compare these solutions to gravitating monopole-antimonopole systems, associating monopole-antimonopole pairs with sphalerons.

Here we consider the effect of gravity on the axially symmetric multisphalerons, and the sphaleron-antisphaleron pairs, chains and vortex rings. We characterize these solutions by two integers, m and n. The Klinkhamer-Manton sphaleron [6] has $m = n = 1$, while the multisphalerons, representing superpositions of n sphalerons, have $m = 1, n > 1$. Sphaleron-antisphaleron pairs are obtained for $m = 2, n = 1,2$, and chains for $m > 2, n = 1,2$, while vortex rings arise for $m > 1, n > 2$. At the same time additional branches of solutions arise, which connect to the generalized Bartnik-McKinnon (BM) solutions [7].

We consider $SU(2)$ Einstein-Yang-Mills-Higgs theory with action

$$S = \int \left\{ \frac{R}{16\pi G} - \frac{1}{2} Tr(F_{\mu\nu} F^{\mu\nu}) - (D_\mu \Phi)^+ (D^\mu \Phi) - \lambda(\Phi^+ \Phi - \frac{v^2}{2})^2 \right\} \sqrt{-g}\, d^4 x , \qquad (1)$$

with curvature scalar R, $su(2)$ field strength tensor $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ie[V_\mu, V_\nu]$, $su(2)$ gauge potential $V_\mu = V_\mu^a \tau_a / 2$, and covariant derivative of the Higgs Φ in the fundamental representation $D_\mu \Phi = (\partial_\mu + ieV_\mu)\Phi$, where G and e denote the gravitational and gauge coupling constants, respectively, λ denotes the strength of the Higgs self-interaction and v the vacuum expectation value of the Higgs field. The action (1) is invariant under local $SU(2)$ gauge transformations U, $V_\mu \to UV_\mu U^+ + \frac{i}{e} \partial_\mu U U^+$, $\Phi \to U\Phi$. The gauge symmetry is spontaneously broken due to the non-vanishing vacuum expectation value of the Higgs field $\langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, leading to the vector and scalar boson masses $M_W = \frac{1}{2} ev$, $M_H = v\sqrt{2\lambda}$. Reexpressing the anomaly term in terms of the Chern-Simons current $K^\mu = \frac{e^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} Tr(F_{\nu\rho} V_\sigma + \frac{2}{3} ieV_\nu V_\rho V_\sigma)$, yields for the fermion charge of a sphaleron solution (in a suitable gauge) $Q_F = \int d^3 r K^0$. To obtain gravitating static axially symmetric solutions, we employ isotropic coordinates. In terms of the spherical coordinates r, θ and φ the isotropic metric reads

$$ds^2 = -fdt^2\frac{h}{f}dr^2 + \frac{hr^2}{f}d\theta^2 + \frac{lr^2\sin^2\theta}{f}d\varphi^2, \tag{2}$$

where the metric functions f, h and l are functions of the coordinates r and θ, only. The z-axis ($\theta = 0, \pi$) represents the symmetry axis. Regularity on this axis requires $h|_{\theta=0,\pi} = l|_{\theta=0,\pi}$. We take a purely magnetic gauge field, $V = 0$, and parametrize the gauge potential and the Higgs field by the Ansatz

$$V_i dx^i = \left(\frac{H_1}{r}dr + (1-H_2)d\theta\right)\frac{\tau_\varphi^{(n)}}{2e} - n\sin\theta\left(H_3\frac{\tau_\theta^{(n,m)}}{2e}\right)d\varphi, \quad V_0 = 0, \tag{3}$$

and

$$\Phi = i\left(\Phi_1\tau_r^{(n,m)} + \Phi_2\tau_\theta^{(n,m)}\right)\frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} . \tag{4}$$

where

$$\tau_r^{(n,m)} = \sin m\theta(\cos n\varphi\tau_x + \sin n\varphi\tau_y) + \cos m\theta\tau_z ,$$
$$\tau_\theta^{(n,m)} = \cos m\theta(\cos n\varphi\tau_x + \sin n\varphi\tau_y) - \sin m\theta\tau_z ,$$
$$\tau_\varphi^{(n)} = (-\sin n\varphi\tau_x + \cos n\varphi\tau_y),$$

n and m are integers, and τ_x, τ_y and τ_z denote the Pauli matrices. The two integers n and m determine the fermion number of the solutions, $Q_F = \frac{n(1-(-1)^m)}{4}$. For vanishing gravity and $m = n = 1$ the Ansatz yields the Klinkhamer-Manton sphaleron[6]. For $n > 1$ or $m > 1$, the functions $H_1,...,H_4,\Phi_1,$ and Φ_2 depend on r and θ, only. These axially symmetric solutions represent gravitating multisphaleron ($m = 1, n > 2$), sphaleron-antisphaleron pair ($m = 2, n = 1$), chain ($m > 2, n = 1$), and vortex ring ($m > 1, n > 2$) configurations as well as mixed configurations. Let us now introduce the dimensionless coordinate x and the dimensionless coupling constant α $x = \frac{e\alpha}{\sqrt{4\pi G}}r, \quad \alpha = \sqrt{4\pi G v}$. The limit $\alpha \to 0$ can be approached in two different ways: 1. $G \to 0$, while the Higgs vacuum expectation value v remains finite (flat-space limit), and 2. $v \to 0$, while Newton's constant G remains finite. These limits are then associated with different branches of solutions. The dimensionless mass M of the solutions is obtained from the asymptotic expansion of the metric function f, $M = \frac{1}{2\alpha^2}\lim_{x\to\infty} x^2\partial_x f = \frac{\mu}{\alpha^2}$.

Let us first briefly recall the new static axially symmetric solutions of Weinberg-Salam theory (in the limit of vanishing Weinberg angle), found recently. We here restrict the discussion to the case of vanishing Higgs mass. The axially symmetric solutions are characterized by two integers, m and n. We have investigated gravitating sphalerons, multisphalerons and sphaleron-antisphaleron systems, which are static and axially symmetric, and characterized by two integers, m and n. Single sphalerons are obtained for $m = n = 1$, multisphalerons for $m = 1$ and $n > 1$, and sphaleron-antisphaleron systems for $m > 1$. Like the electroweak sphaleron these new solutions are unstable, corresponding to saddle points.

In the presence of gravity, from each of these flat space solutions, a branch of gravitating solutions emerges and evolves smoothly with increasing gravitational coupling constant α up to a maximal value $\alpha_{\max}$. There it merges with a second branch, higher in energy, which extends backwards to $\alpha = 0$. In the limit, the Higgs vacuum expectation value vanishes, and the limiting solutions correspond to pure EYM solutions (after rescaling).

For larger values of the Higgs mass, the flat space solutions are no longer uniquely specified by the integers m and n. Instead bifurcations appear, giving rise to further branches and types of configurations. As for the monopole-antimonopole systems, we therefore expect a plethora of gravitating solutions at large scalar coupling. Furthermore, for very large values of the Higgs mass also bisphalerons or `deformed' sphalerons are present, which do not exhibit parity reflection symmetry.

Comparing these gravitating sphaleron, multisphaleron and sphaleron-antisphaleron solutions, based on a doublet Higgs field, with the monopole-antimonopole solutions, obtained with a triplet Higgs field, we find precisely the same pattern of branches of solutions, when we compare sphalerons and sphaleron-antisphaleron systems characterized by m and n, with monopole-antimonopole systems characterized by $2m$ and n. Interestingly, in the case $n = 4$, the scaled mass of both types of solutions even almost coincides.

Monopole-antimonopole systems can rotate, when they carry no magnetic charge. It therefore appears interesting to consider also rotating sphaleron-antisphaleron systems. Moreover, monopole-antimonopole systems can be endowed with a black hole at their center, as shown explicitly already for the monopole-antimonopole pair [2, 8]. Sphaleron-antisphaleron systems with black holes are thus expected to exist as well.

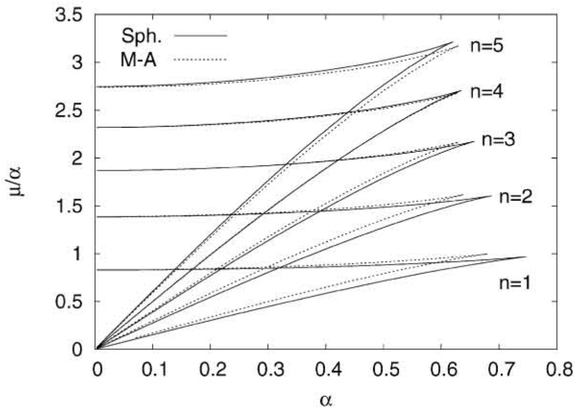

Figure 1: *Scaled mass μ/α versus coupling constant α for the single sphaleron ($m=1, n=1$) and multisphaleron ($m=1, n=2,...,5$) solutions; for comparison the mass of the monopole-antimonopole pair ($m=2, n=1,2$) and vortex ring ($m=2, n=3,5$) solutions is also shown.*

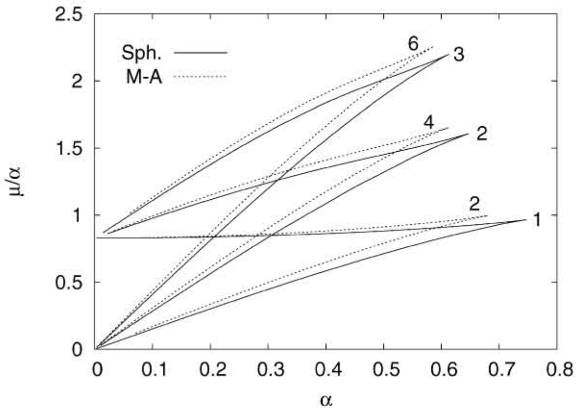

Figure 2: *Scaled mass μ/α versus coupling constant α for single sphaleron ($m=1, n=1$), sphaleron-antisphaleron pair ($m=2, n=1$), and chain ($m=3, n=1$) solutions; for*

comparison the mass of the monopole-antimonopole pair ($m = 2, n = 1$), and chain ($m = 4, n = 1$), ($m = 6, n = 1$) solutions is also shown.

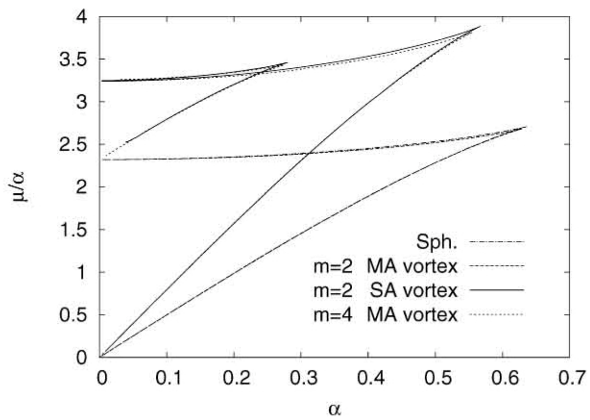

Figure 3: *Scaled mass μ/α versus coupling constant α for single sphaleron ($m = 1, n = 4$ and sphaleron-antisphaleron vortex ring ($m = 2, n = 4$) solutions; for comparison the mass of monopole-antimonopole ($m = 2, n = 4$) and monopole-antimonopole ($m = 4, n = 4$) vortex ring solutions is also shown.*

Acknowledgement

R.I. and U.Kh. gratefully acknowledges support by the Volkswagen Foundation, and B.K. support by the DFG.

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