EXISTENCE OF WEAK SOLUTIONS OF THE g-KELVIN – VOIGHT EQUATION

A. Okay Çelebi

Yeditepe University, Istanbul, Turkey, *acelebi@yeditepe.edu.tr*

In this talk we will present the existence of weak solutions for the g-Kelvin-Voight equations. The results given here are published in [12].

The incompressible Newtonian fluid flows are described by Navier-Stokes equations given by

$$\frac{\partial U}{\partial t} - v\Delta U + U \cdot \Delta U + \Delta P = F \text{ in } \Omega$$
$$\nabla \cdot U = 0, \tag{1}$$

where $\Omega \subset R^3, \Delta$ is the Laplace operator v and F are the kinematic viscosity and the external force respectively. In the above equation the velocity vector U and the pressure P are unknown. These equations are studied extensively, we refer the reader to [4, 5, 13, 19], for some of the literature.

The idea of investigating 3-dimensional nonlinear equation in thin domains was introduced by Hale and Raugel [6, 7] in $\Omega_2 \times (0, \varepsilon)$ where $\Omega_2 \subset R^2, 0 < \varepsilon < 1$. Afterwards Raugel and Sell studied the Navier-Stokes equations in thin domains [17]. They use vertical mean operator M which decompose every function U on Ω_ε into the sum of a function $MU = v(x_1, x_2)$ and a function $(1 - M)U = w(x_1, x_2, x_3)$ where v corresponds to the solution reduced 3D Navier-Stokes equations. Since v depends on only 2 spatial variables, it is possible to use better estimates. In the case of a varying bottom Camassa, Holm and Levermore [2] have derived a family of shallow water equations which model the circulation of a fluid in a large shallow basin. In the derivation of these models they have used the weighted divergence condition. If the bottom is flat, divergence free models are obtained [2]. Assuming that the function representing the topography of the bottom is nondegenerate, they have obtained the long-time effects of the bottom topography for the lake and great lake equations. Levermore, Oliver and Titi [14, 15], have obtained global existence and uniqueness of lake and great lake equations. Roh [18] has employed the technique of Hale, Raugel and Sell in [6, 7, 17] to the Navier-Stokes equations to the domains $\Omega_2 \times (0, g)$. Some properties of the solutions are given in [1, 11, 18]. Kelvin-Voight equations have also been studied extensively, see for example [8, 16]. Recently Cao, Lunasin, Titi [3] have obtained the global regularity of the inviscid version of 3D Kelvin-Voight model. The Gevrey regularity of the global attractor and determining modes for the 3D Navier-Stokes-Voight equations (Kelvin-Voight equations) are given in [9, 10].

The motion of visco-elastic fluid and the basic properties of visco-elastic materials can be described by Kelvin-Voight equations which are given by

$$\frac{\partial U}{\partial t} - v\Delta U - \alpha \Delta U_t + U \cdot \Delta U + \Delta P = F \text{ in } \Omega$$
$$\nabla \cdot U = 0$$

in 3D.

The change of variables, $y_1 = x_1,\ y_2 = x_2,\ y_3 = x_3 g(x_1, x_2)$ maps Ω_3 onto Ω_g in 3D where $\Omega_3 = \Omega \times [0,1], \Omega_g = \Omega \times [0, g]$ and U is a function of $y = (y_1, y_2, y_3) \in \Omega_g$ [18]. Then using the operators $M\ ;\ I - M$, which are given in [6, 7, 17] we obtain the g-Kelvin -

Voight equations

$$\begin{aligned}&\frac{\partial u}{\partial t}-\frac{\nu}{g}(\nabla\cdot g\nabla)u+\frac{\nu}{g}(\nabla g\cdot\nabla)u-\frac{\alpha}{g}(\nabla\cdot g\nabla)u_t+\\&+\frac{\alpha}{g}(\nabla g\cdot\nabla)u_t+u\cdot\nabla u+\nabla p=f(x)\end{aligned}\tag{2}$$

$$\nabla\cdot(gu)=0 \text{ in } \Omega\times[0,T]\tag{3}$$

$$u(x,0)=u_0(x) \text{ in } \Omega\tag{4}$$

$$u=0 \text{ in } \partial\Omega\times[0,T]\tag{5}$$

where $\Omega\subset R^2$ is a bounded domain $\nu>0, \alpha>0$ and Δ_g, g - Laplacian operator which is defined by

$$-\Delta_g u=-\frac{1}{g}(\nabla\cdot g\nabla)u=-\Delta u-\frac{1}{g}(\nabla g\cdot\nabla)u\,.$$

It is trivial that g-Kelvin-Voight model is reduced to the Kelvin-Voight model in *2D* when the thickness is uniform.

In our article [12] we have discussed the existence and uniqueness of the solutions of (2)-(5). Our main results may be stated as:

Theorem 1. If $f\in L^2(\Omega,g), u_0\in V_g$ and g satisfy

(i) $g(x)\in C^\infty(\Omega)$,

(ii) $\Delta g=0$,

(iii) $0<n\le g(x)\le N$ where $n=n(g)$ and $N=N(g)$ are constants for all $x\in\Omega$,

then there exists at least one weak solution to the problem (2)-(5).

Theorem 2. Under the hypothesis of Theorem 1, the weak solution of the problem (2)-(5) is unique.

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