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TWO EVENTOLOGICAL ASYMPTOTIC THEOREMS

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Two asymptotic theorems for a Gibb's and an opposite Gibb's eventological distributions (E-distributions) are formulated and proved. From the point of view of the Eventology any event can be considered as a dual event consisting of two events: an event-perception and an event-activity. So an event connected with some goods at the commodity market can be also considered as dual event consisted of two events: event-demand and event-supply. The event-demand plays a role of an event-perception, and the event-supply plays a role of an event-activity. The Gibb's E-distribution is used to model a value of demand, and the opposite Gibb's E-distribution is used to model a value of supply. With aspiration of parameters of these distributions to extreme values distributions accept the asymptotic kinds, proved by two theorems. Such approach allows receive the dual E-model of market considering a joint market consumer's and manufacturer's behaviors.

In the work [2] is given an eventological substantiation of a classical model of market where the consumer (buyer) and the manufacturer (seller) possess opposite market properties which can be shown if money and goods would be changed places:

- the consumer of the goods for money is a manufacturer of money who offers money for the goods;
- the manufacturer of the goods for money is a consumer of money which asks money for the goods.

Opposite market properties of the goods, of the consumer and the manufacturer allow to use the Gibb's E-model consumer of goods for the money [1] to make an opposite Gibb's E-model manufacturer of goods for the money. A demand of the goods for money is an event which we shall name an event-demand. It is the demand in the standard understanding. The event-demand corresponds to the event-perception in the [3]. A demand of money for the goods is an event too, which is standard to consider as the supply. The event-supply corresponds to the event-activity also considered in the [3]. Thus, two events are connected with each goods $x \in \mathrm{X}$: the event-demand $x^{\downarrow}$ and the event-offer $x^{\uparrow}$ which form two sets:

$$\mathrm{X}^{\downarrow} = \{x^{\downarrow} : x \in \mathrm{X}\},\ \mathrm{X}^{\uparrow} = \{x^{\uparrow} : x \in \mathrm{X}\}.$$

In the work [1] is shown, that the set of events-demands has the Gibb's E-distribution

$$p^{\downarrow}(X^{\downarrow}) = \frac{1}{Z_{p_*^{\downarrow}}} \exp\{-\beta V^{\downarrow}(X^{\downarrow})\} p_*^{\downarrow}(X^{\downarrow}),\ X \subseteq \mathrm{X},\quad \beta \geq 0,$$

which minimizes a relative entropy of the E-distribution $p^{\downarrow}$ concerning the E-distribution $p_*^{\downarrow}$ (an eventological analogue of the Boltzmann's *H*-theorem). Parameters of the Gibb's E-distribution make sense:

- β is the non-negative size equal to an average from opposite purchasing capacity of the given consumer which characterizes its own E-distribution of demand in whole for all set of the goods X , i.e. it is a consumer's numerical characteristic;
- $V^{\downarrow}(X^{\downarrow})$ is a function of value of a subset $X^{\downarrow}$ of events-demands, the non-negative limited function on the $2^{\mathrm{X}^{\downarrow}}$;

- $p_*^{\downarrow}(X^{\downarrow})$ is some fixed E-distribution on $2^{\mathrm{X}^{\downarrow}}$, which is interpreted as the own E-distribution of demand of the consumer;
- $Z_{p_*^{\downarrow}} = \sum_{X \subseteq \mathrm{X}} \exp\{-\beta V^{\downarrow}(X^{\downarrow})\} p_*^{\downarrow}(X^{\downarrow})$ is the multiplier providing a probabilistic normalizing of the E-distribution of demand;

The Gibb's E-distribution can be written in more compact form:

$$p^{\downarrow}(X^{\downarrow}) = \frac{1}{Z_{p_*^{\downarrow}}} B_{\beta}^{\downarrow}(X^{\downarrow}) p_*^{\downarrow}(X^{\downarrow}) = \frac{1}{Z_{p_*^{\downarrow}}} G_{\beta}^{\downarrow}(X^{\downarrow}), \ X \subseteq \mathrm{X}, \quad \beta \geq 0,$$

where $B_{\beta}^{\downarrow}(X^{\downarrow})$ is a Boltzmann's factor, and $G_{\beta}^{\downarrow}(X^{\downarrow}) = B_{\beta}^{\downarrow}(X^{\downarrow}) p_*^{\downarrow}(X^{\downarrow})$ is a Gibb's factor.

In the work [4] is shown, that a set of events-supply has an opposite Gibb's E-distribution

$$p^{\uparrow}(X^{\uparrow}) = \frac{1}{Z_{p_*^{\uparrow}}} \exp\{\gamma V^{\uparrow}(X^{\uparrow})\} p_*^{\uparrow}(X^{\uparrow}), \ X \subseteq \mathrm{X}, \quad \gamma \geq 0,$$

which minimizes the relative entropy of the E-distribution $p^{\uparrow}$ concerning to the E-distribution $p_*^{\uparrow}$ (the eventological analogue of the Boltzmann's *H*-theorem).

Parameters of the opposite Gibb's E-distribution make sense:

- γ is the non-negative size equal to an average from return productive ability of the given manufacturer which characterizes its own E-distribution of supply as a whole for all set of the goods X, i.e. it is a consumer's numerical characteristic;
- $V^{\uparrow}(X^{\uparrow})$ is a function of value of a subset $X^{\uparrow}$ of events-supply, the non-negative limited function on the $2^{\mathrm{X}^{\uparrow}}$;
- $p_*^{\uparrow}(X^{\uparrow})$ is some fixed E-distribution on $2^{\mathrm{X}^{\uparrow}}$, which is interpreted as the own E-distribution of supply of the manufacturer;
- $Z_{p_*^{\uparrow}} = \sum_{X \subseteq \mathrm{X}} \exp\{\gamma V^{\uparrow}(X^{\uparrow})\} p_*^{\uparrow}(X^{\uparrow})$ is the multiplier providing probabilistic normalizing of the E-distribution of supply.

The opposite Gibb's E-distribution can be written in more compact form:

$$p^{\uparrow}(X^{\uparrow}) = \frac{1}{Z_{p_*^{\uparrow}}} B_*^{\uparrow}(X^{\uparrow}) p_*^{\downarrow}(X^{\uparrow}) = \frac{1}{Z_{p_*^{\uparrow}}} G_*^{\uparrow}(X^{\uparrow}), \ X \subseteq \mathrm{X}, \quad \gamma \geq 0,$$

where $B_{\gamma}^{\uparrow}(X^{\uparrow})$ is a Boltzmann's antifactor, and $G_{\gamma}^{\uparrow}(X^{\uparrow}) = B_{\beta}^{\downarrow}(X^{\uparrow}) p_{\gamma}^{\uparrow}(X^{\uparrow})$ is a Gibb's antifactor.

At aspiration of purchasing and/or productive ability to the least or to the greatest values E-distributions of supply and demand aspire to asymptotic E-distributions which kinds are defined by two theorems.

As in this work market's E-models are considered, processes of aspiration of parameters of E-distributions to zero or to infinity will be called by the most suitable market terms "freezing" or "warming up". In the work [1] the same processes were called "hardening" and "fusion" (terms have been borrowed from statistical physics) and the theorem of a kinds asymptotic Gibb's E-distributions has been proved. The eventological generalization of analogue of the Boltzmann's *H*-theorem [4] has naturally led to idea of similar generalization of the theorem of "freezing" and "warming up". In the given work theorems of "freezing" and "warming up" of the Gibb's and of the opposite Gibb's E-distributions of supply and demand accordingly are formulated.

Theorem 1 ("freezing" and "warming up" of the Gibb's E-distribution of demand).
Let $\mathrm{X}^{\downarrow}$ *be a set of events-demands having the Gibb's E-distribution*

$$p^{\downarrow}\left(X^{\downarrow}\right)=\frac{1}{Z_{p_*^{\downarrow}}}B_{\beta}^{\downarrow}\left(X^{\downarrow}\right)p_*^{\downarrow}\left(X^{\downarrow}\right)=\frac{1}{Z_{p_*^{\downarrow}}}G_{\beta}^{\downarrow}\left(X^{\downarrow}\right),\ X\subseteq \mathrm{X},\quad \beta\geq 0,$$

which is defined by the Boltzmann's factor $B_{\beta}^{\downarrow}\left(X^{\downarrow}\right)$ *with the own E-distribution of events-demands* $p_*^{\downarrow}\left(X^{\downarrow}\right)$*, i.e. it is defined by the Gibb's factor* $G_{\beta}^{\downarrow}\left(X^{\downarrow}\right)=B_{\beta}^{\downarrow}\left(X^{\downarrow}\right)p_*^{\downarrow}\left(X^{\downarrow}\right)$*. Then*

1. *at freezing of the Gibb's E-distribution* $(\beta\to\infty)$ *it aspires in a limit either to zero, or to the value proportional (only on subsets from* M_0*) to the own distribution:*

$$\lim_{\beta\to\infty}p^{\beta}\left(X^{\downarrow}\right)=\begin{cases}p_*^{\downarrow}\left(X^{\downarrow}\right)/\,p_{*0}^{\downarrow}, & X\in \mathrm{M}_0,\\ 0, & \textit{иначе},\end{cases}$$

here

$$\mathrm{M}_0=\left\{X:\quad V^{\downarrow}\left(X^{\downarrow}\right)=\min_{Y}V^{\downarrow}\left(X^{\downarrow}\right)\right\}\subseteq 2^{\mathrm{X}}$$

is a set of subsets on which function of value of events-demands $V^{\downarrow}\left(X^{\downarrow}\right)$ *accepts the minimal value, and*

$$p_{*0}^{\downarrow}=\sum_{X\in \mathrm{M}0}p_*^{\downarrow}\left(X^{\downarrow}\right)$$

is the own probability of a minimum of function of valu;

2. *and at "warming up" of the Gibb's E-distribution* $(\beta\to 0)$ *it aspires in a limit to its own distribution:*

$$\lim_{\beta\to 0}p^{\beta}\left(X^{\downarrow}\right)=p_*^{\downarrow}\left(X^{\downarrow}\right),\ \left(X^{\downarrow}\right)\in 2^{\mathrm{X}}.$$

Theorem 2 ("freezing" and "warming up" of the E-distribution of supply).
Let $\mathrm{X}^{\uparrow}$ *is a set of events-supply with the opposite Gibb's E-distribution*

$$p^{\uparrow}\left(X^{\uparrow}\right)=\frac{1}{Z_{p_*^{\uparrow}}}B_{\gamma}^{\downarrow}\left(X^{\uparrow}\right)p_*^{\uparrow}\left(X^{\uparrow}\right)=\frac{1}{Z_{p_*^{\uparrow}}}G_{\gamma}^{\uparrow}\left(X^{\uparrow}\right),\ X\subseteq \mathrm{X},\quad \gamma\geq 0,$$

which is defined by the Boltzmann's antifactor $B_{\gamma}^{\uparrow}\left(X^{\uparrow}\right)$ *and by its own E-distribution of events-supply, i.e. the Gibb's antifactor* $G_{\beta}^{\uparrow}\left(X^{\downarrow}\right)=B_{\beta}^{\downarrow}\left(X^{\downarrow}\right)p_*^{\downarrow}\left(X^{\downarrow}\right)$*. Then*

1. *at freezing of the opposite Gibb's E-distribution* $(\gamma\to\infty)$ *it aspires in a limit either to zero, or to the value proportional (only on subsets from* M_1*) to its own distribution:*

$$\lim_{\gamma\to\infty}p^{\gamma}\left(X^{\uparrow}\right)=\begin{cases}p_*^{\uparrow}\left(X^{\uparrow}\right)/\,p_{*0}^{\uparrow}, & X\in \mathrm{M}_1,\\ 0, & \textit{иначе},\end{cases}$$

Here

$$\mathrm{M}_1=\left\{X:\quad V^{\uparrow}\left(X^{\uparrow}\right)=\max_{Y}V^{\uparrow}\left(X^{\uparrow}\right)\right\}\subseteq 2^{\mathrm{X}}$$

is a set of subsets on which function of value of events-supply $V^{\uparrow}\left(X^{\uparrow}\right)$ *accepts the maximal value, and:*

$$p_{*0}^{\uparrow}=\sum_{X\in \mathrm{M}0}p_*^{\uparrow}\left(X^{\uparrow}\right)$$

is the own probability of a maximum of function of value $V^{\uparrow}$*;*

2. *and at "warming up" of the opposite Gibb's E-distribution* $(\gamma\to 0)$ *it aspires in a limit to the own distribution*

$$\lim_{\gamma\to 0}p^{\gamma}\left(X^{\uparrow}\right)=p_*^{\uparrow}\left(X^{\uparrow}\right),\ \left(X^{\uparrow}\right)\in 2^{\mathrm{X}}.$$

Two theorems of "freezing" and "warming up" of the Gibb's and the opposite Gibb's E-distributions of supply and demand allow to model behaviors of participants of the market at their extreme values of purchasing and productive abilities accordingly.

If an interpretation of asymptotic Gibb's distribution of the consumer is habitually enough: at reduction of purchasing capacity (at "freezing") the E-distribution of the consumer concentrates on the minimal consumer's basket proportionally to its own E-distribution of demand, and at increase in purchasing capacity (at "warming up") the E-distribution of the consumer aspires to its own E-distribution of demand. So an interpretation of the second theorem of "freezing" and "warming up" of the manufacturer consists in the following. Though the proof of the theorem 2 is similar to the proof of the theorem 1, but in sense of market interpretation there is one essential distinction between them. If in the theorem 1 "freezing" it is interpreted as a concentration of E-distribution on the minimal basket of demand 2 speech should go to interpretations of the theorem about a concentration opposite Gibb's E-distributions to maximal "portfolio" of supply.

Literature

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