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ABOUT ONE APPROACH TO UTILITY FUNCTION IDENTIFICATION

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Traditional models of the demand forecasting do not consider non-metrical factors, reasoning this, mostly with the consumer behavior. Significant impact of these factors on consumers' demand is obvious. In essence, their existence do not allow considering time series model with the highest base as an acceptable. Therefore, to understand the basis of the demand and receive her adequate curve, it is necessary to research one of the main, hardly formulated, characteristics of the consumer behavior, which is utility (level of satisfaction) from the consuming goods and services set.

During the utility research it is necessary to consider that, initially, consumers have full information, related to consumer decisions (information about the goods assortment and the capacity of each to provide necessary utility, about prices, and about personal income), moreover, consumers are able to range all conceivable set of goods on the base of the ability of each set to provide utility. Consumer always tries to maximize her level of satisfaction or, as it called by economists, *utility*, which defines as individual perception of satisfaction from any consuming goods and services set. From mathematical point of view consumer behavior observes in the selection of certain point from the "space of goods". If we will assume that existence of *n* goods, then quantity of each from them, acquired by consumer, characterizes with the *set of goods* $X = (x_1, x_2, ..., x_n)$, where x_i is the quantity of *i-th* good (service), acquired by consumer. Let us assume that all goods have a random divisibility feature, i.e. could be acquired in any non-negative quantity of each. Therefore, all possible sets are formed a closed convex vector space $C = \left\{X = (x_1, ..., x_n) \mid x_i \geq 0, i = \overline{1,n}\right\}$, which presents the weak regulation and is uninterrupted [1]. Last expression allows us to claim, that on this space of goods exists uninterrupted real function $U(\cdot)$, called as *utility function*, for which $U(X) \geq U(Y)$ inequality occurs at the preference of X over Y good. If U presents some kind of utility index, then value of the index depends on the quantity of the consumed x_i goods.

In this paper neuro-fuzzy approach to the utility function formulization is established on the implementation of uncertain logical deduction method and identification of neurons. Basis for the assumption is that main values of the consumer problem (consumer income and goods' market price) receive interval values, and values of suitable utility functions have conditional nature. Let us calculate the appropriate utility functions for selective assortments of goods and services, which satisfy the settled restrictions on incomes and market prices.

Further, we will accept that space of various sets of goods and services for consumer market is closed and convex vector hyperspace *C*. Then, on multitude of the restricted consumer incomes from different categories and from market prices for goods considered from the settled as an interval values that identifies the utility function from the consuming set.

To identify utility in each certain case it is necessary to consider that in dynamic conditions of the market and in competitive environment price of goods and consumer incomes can not stay fixed/stable. Generally, they varied in certain frames and in short-term period defined as averaged values. Finally, this will result in an error, which sometimes does not provide required adequate solution. Therefore, it is advisable to use linguistic variables to describe market prices of goods (services) and consumer incomes, values of which can be uncertain therm-sets [2]. Moreover, the concept of '"utility" by itself belongs to the qualitative category and therefore, linguistic variable can be used as a criteria of utility, which receives uncertain values.

Functional dependence of the utility defines by the way of formation of the uncertain logical rules, where linguistic variable - "utility" accepted as endogenous value, however,

"consumer income" and "market prices" of goods and services are considered as exogenous linguistic values. Consequently, our task is the formation the family of uncertain levels of the utility from the set of consuming goods and services, defuzzyficated values that will be considered conditional alternative values for desired function of utility.

Let us assume that we have to deal with the segmented market, where space of possible set of consuming goods and definite services present the closed convex and uninterrupted space $C = \left\{X = (x_1,...,x_n) \mid x_k \geq 0, k = \overline{1,n}\right\}$, where x_k represents conditional quantity of k goods (services), acquired by consumer. Further, let us divide consumers on q linguistic categories by levels of the uncertain incomes of $\tilde{r}_i$ ($i = \overline{1,q}$), part of which they are ready or able to spend on a purchase of goods from this segment. Then, considering the fuzzy value of $\tilde{p}_k$ price for goods from the X set, functional dependence of utility will represent a set of uncertain rules of m [3]:

$$\text{if } I = \tilde{r}_i \text{ and } P_1 = \tilde{p}_1 \text{ and } P_2 = \tilde{p}_2 \text{ and } \ldots \text{ and } P_1 = \tilde{p}_n, \text{ then } U = \tilde{u}_j, \quad (1)$$

where: $\tilde{r}_i$ ($i = \overline{1,q}$) – is uncertain i is the level of the consumer incomes; $\tilde{p}_k$ ($k = \overline{1,n}$) – is uncertain level of prices for a k good from set of X; $\tilde{u}_j$ ($j = \overline{1,5}$) – is uncertain value of j utility function.

Now, let us choose list of goods and services as a consumer market segment, usage of which is necessary and relatively sufficient for normal humankind development. These goods and services may be the following, to which consumer spends considerable part of her income in one month, to receive all expected from goods for self satisfaction.

a_1 – bakery goods in a price range 0.1÷0.5 AZN for a unit;
a_2 – meat foods in price range 2.00÷10 AZN for a unit;
a_3 – dairy produces in price range 0.70÷1.50 AZN for a unit;
a_4 – fruits and vegetables in price range 0.10÷2 AZN for a unit;
a_5 – confectionary in price range 0.50÷10 AZN for a unit;
a_6 – beverage foods in price range 0.40÷10 AZN for a unit;
a_7 – tobacco goods in price range 0.30÷1.50 AZN for a unit;
a_8 – consumer and cultural-welfare facility services in price range 1.00÷50 AZN for a unit;
a_9 – freight/transport services in price range 0.10÷2.50 AZN for a unit.

As it is seen from the list, unit prices of goods may vary in appropriate ranges. Hence, for each k good it may have uncertain values like, $\tilde{p}_k^{bot}$ – "low", $\tilde{p}_k^{avr}$ – "average" and $\tilde{p}_k^{top}$ – "high". Changes in price levels (ranges) in to the negative or positive side makes consumer accordingly "richer" or "poorer" in the frame of the stable income. This, in one turn, gives her appropriately opportunity to satisfy her needs "more" or "less" with the consuming goods. Further, considering the offered monetary equivalent of the lowest subsistence wage, we will divide consumers according to their income levels into the following categories:: $\tilde{r}_1$ – with "low" income level (70÷150 AZN), $\tilde{r}_2$ – with "under the average" income level (151÷300 AZN), $\tilde{r}_3$ – with "average" income level (301÷500 AZN), $\tilde{r}_4$ – with "over the average" income level (501÷900 AZN) and $\tilde{r}_5$ – with "high" income level (901 AZN and higher).

To formation the functional dependencies between uncertain category of the "utility" from the consuming sets, which receives uncertain values as $\tilde{u}_j$ ($j = \overline{1,5}$): "low", "under the average", "average", "over the average" and "high", and selected uncertain values of the "price for goods" and the "income level" on similarity of (1), 15 uncertain linguistic rules have been established. Evaluation of these rules is possible to implement in the MATLAB/Fuzzy Logic Toolbox software environment, where for the fuzzyfication of the inputs of uncertain therm-sets Gausses function could be selected and defuzzyfication of the uncertain outputs (uncertain levels of utility) is possible on the base of the centrode method. Particularly, this is possible in presence of the fixed prices for goods and services from the considered list, by settling

defuzzyficated levels (values) of utility from consuming of 15 random sets by the consumers with the different levels of income. (Table.1).

Table.1

Utility from consuming the random set of goods and services

Consu-mer income (AZN)	Random set of goods and services									Utility
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	
88,09	14,01	2,00	20,00	20,0	9,00	3,00	2,0	1,00	10,00	0,261
719,14	14,98	6,00	20,00	28,0	12,00	13,00	12,0	20,00	20,00	0,590
840,62	16,22	6,00	20,00	50,0	15,00	30,00	15,0	66,00	60,00	0,662
295,51	14,52	5,00	16,00	20,0	15,00	15,00	15,0	10,00	20,00	0,420
852,10	16,32	6,00	16,00	47,0	15,00	25,00	15,0	60,00	50,00	0,707
652,51	18,00	6,00	16,00	40,0	15,00	25,00	15,0	46,00	37,00	0,582
488,89	15,73	6,00	16,00	34,0	15,00	20,00	15,0	16,00	21,00	0,493
609,52	16,14	5,00	13,00	15,0	15,00	16,00	15,0	10,00	25,00	0,607
324,46	14,03	6,00	15,00	15,0	15,00	13,00	14,0	3,00	5,00	0,418
705,52	16,81	6,00	15,00	15,0	15,00	13,00	14,0	20,00	25,00	0,612
104,54	13,70	4,00	5,00	5,0	5,00	5,00	5,0	0,50	4,00	0,307
931,37	17,38	6,00	25,00	50,0	40,00	50,00	20,0	85,00	83,00	0,721
198,65	18,61	5,00	25,00	15,0	15,00	15,00	15,0	1,00	15,00	0,417
187,36	16,70	5,00	12,00	10,0	10,00	10,00	10,0	0,50	10,00	0,363
271,30	15,20	6,00	12,00	11,0	10,00	13,00	15,0	2,00	15,00	0,391

The received random selection of 15 "set – utility" pairs based on uncertain rules (1) application can be used for approximation of utility function on the base of feedforward neural network with the single nonlinear "concealed" cover. Particularly, neural network which is generated in MATLAB/Neural Network Toolbox software environment (Fig. 2) after successful training (Fig. 1) enables to receive completely adequate values for utility function.

Training of the neural networks applied on the basis criterion of the mean-square deviation in the comparator unit (Fig. 1)

$$E = \frac{1}{2}\sum_{j=1}^{15}\left(u_j - u_j^{net}\right)^2 \qquad (2)$$

and gradient algorithm of "error backpropagation" at the training block. Selection of the acceptable indentified error e after the trainings provides possibility to receive optimal values for input and output weights of connections and "thresholds" of nonlinear neurons from "concealed" cover of neural network.

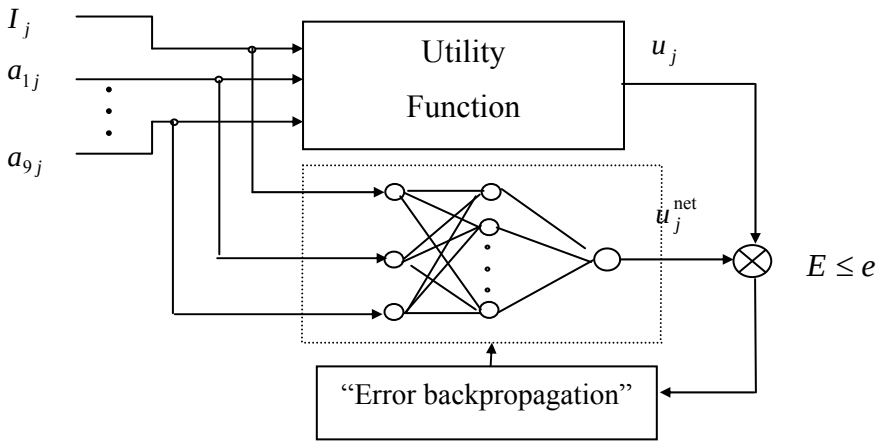

Fig. 1 Neural identification function of the utility

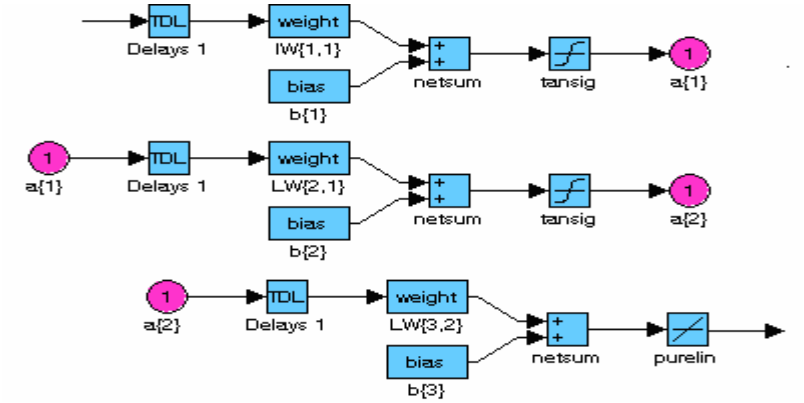

Fig. 2 Feedforward Neural Network in MATLAB notation

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