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ON STAGES JOINT OF MODELING AND DECISION MAKING ON CONTROLLING TECHNOLOGICAL PROCESSES UNDER CONSTRAINTS

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In order to investigate technological processes and to control them an apparatus of statistical modeling, which has made a good showing provided that the drift of mathematical models is small enough, is widely used. In case of complex, substantially non-linear processes, one has to solve the problem of selection of an adequate mathematical model. Taking into account that a model which is sufficiently adequate for the entire domain of allowable parameters of the process is far from being always satisfactory for some chosen local domain, it is suggested to use parametrical identification of the mathematical model of the process with its sequential refinement, which is carried out in the process of optimization of the process' regimes. Two variants of the decision-making procedure, which represent iterative processes, combining the optimization of control parameters with the over determination of the coefficients of the mathematical model, are suggested. The former consists in that an operation of full intercepting of non-local observed information is used at the each iteration of the optimization of control parameters and of identification of the model. The latter allows using the whole set of statistical data, but each observation obtains the weight depending on its remoteness from the found current optimum point obtained for the model, which was built on the previous step.

Let the controlled static (stationary object) be characterized by the vector of input parameters $x \in X \subset E^r$, the values of which are determined independently on the object itself; X is the given set of possible values of the input parameters of the object; $u \in U \subset E^n$ is a vector of control (optimized) parameters (regimes), where U is a set of allowable values of the controls, which as a rule has simple structure; $p \in E^L$ is a vector of parameters of the object, the values of which are determined merely by the properties of the object itself; $y \in E^{m+1}$ is a vector of output parameters depending on the above mentioned parameters.

The dependences

$$y_i = y_i(u; x^i, p_i), \quad p_i \in E^{l_i}, \quad i = 0,...,m, \qquad L = \sum_{i=0}^{m} l_i , \tag{1}$$

where $y_i(.,.,.), i = 0,1,...,m$ are the given $(n + r + l_i)$-dimensional functions from the class of functions chosen at the first stage. These functions describe the link between the output y and input x, the control parameters u and the parameters of the object p and determine the mathematical model of the object (1).

Under the decision-making problem while controlling the object (optimization of regimes) we will understand the optimization of someone output parameter at the specifically given value of the vector of the input parameters $\bar{x} \in X$ and the bounded values of the other output parameters provided that the parameters of the object are already determined. For instance:

$$y_0(u; \bar{x}, p_0) \to \min_{u \in U} , \tag{2}$$

$$y_i(u; \bar{x}, p_i) \le c_i, \ i = 1,...,m , \tag{3}$$

$$u_j^{\min} \le u_j \le u_j^{\max}, \ j = 1,...,n , \tag{4}$$

where c_i, $u_j^{\min}$, $u_j^{\max}$, $i = 1,...,m,$ $j = 1,...,n$ are the given values determined by technical, economical, technological and planned factors; $\bar{x} \in X$ is the given allowable value of the vector of input parameters; $p = (p_0, ..., p_m)$ is the beforehand determined vector of the parameters of the object [2].

Suppose that

$$(\tilde{U}, \tilde{X}, \tilde{Y}, \tilde{\Gamma}) = \left\{ (\tilde{u}^i, \tilde{x}^i, \tilde{y}^i, \tilde{\gamma}^i) : u^i \in E^n, x^i \in E^r, y^i \in E^{m+1}, 0 \le \tilde{\gamma}^i \le 1,\ i = 1, ..., N \right\} \quad (5)$$

is a set of values of control, input and output parameters, obtained while observing over the process of functioning; $\tilde{\gamma}^i,\ i = 1, ..., N$ are weight parameters determined by the degree of certainty of the measured values of the parameters of the i^{th} observation; N is the number of the carried out independent observations over the state of the object at different input and control parameters.

As it is known, the classical method of decision-making problem on controlling an object consists in building a mathematical model for the object in the beginning, i.e. in determining the parameters, p and in forming the statement of the problem (2) - (4) with the use of observations (5). Then, each time for the current (given) vector of the values of the input parameters $\bar{x} \in X$ with the purpose of making a decision on controlling the object, the optimization problem (2) - (4) is solved and the allowable optimal control u^* corresponding to the input parameter $\bar{x}$ is determined.

Enter the notion of an informational model of the object, which includes:

a) the set of observations $(\tilde{U}, \tilde{X}, \tilde{Y}, \tilde{\Gamma})$;
b) the view of dependences (class of functions) $y_i = y_i(u; x, p)$ determined accurate within the parameters p;
c) the allowable sets of values of the input and control parameters X, U, determined by the values of $c_i, u_j^{\min}, u_j^{\max}, i = 1, ..., \overline{m},\ j = 1, ..., n.$

The base of the automated system of decision-making on controlling the object in the proposed approach is reserved all the time for the informational model, not for the mathematical model of the process.

The k^{th} iteration of some used numerical optimization method of solving the problem (2), (3) we will represent in the form:

$$u^{k+1} = P_U(u^k + \alpha S^k), \ \ k = 0, 1, ... \quad (6)$$

where $S^k = S^k(y(u^k, \bar{x}, p^k))$ is the direction of one-dimensional minimum search, determined first of all by the method of conditional optimization, as well as by all functions participating in the statement of the problem (2), (3); $P_U(u)$ is the operator of projection on the allowable set of controls U, determined by the constraints (4) [2].

The values of the parameters p^k at each iteration (6) in both variants of the suggested approach we will determine reasoning from the informational model, taking into account the current values of the control vector u^k and the input vector $\bar{x}$, with the use of regression analysis method and of mean-square deviation criteria [4].

Consider the first variant of the suggested approach.

Enter the designation: $J = \{1, ..., M\}$. Suppose that $J(u^k, \delta)$ is a subset of indexes of the observations, which are away from the current control vector u^k and from the values of input parameters at the distance not more than δ, i.e.:

$$J(u^k, \bar{x}, \delta) = \left\{ j = J : \left\| \tilde{u}^j - u^k \right\|_{E^n} < \delta, \quad \left\| \tilde{x}^j - \bar{x} \right\| \le \delta \right\}.$$

In the suggested first variant of the approach, the current values of the parameters of the model at each iteration (6) we obtain from the least-squares method with the following target functional:

$$\sum_{j \in J(u^k, x, \delta))} \gamma^j \left[y(\tilde{u}^j; \tilde{x}^j, p) - y^j \right]^2 \to \min_p . \tag{7}$$

The functions $y_i = y_i(u; x, p)$ are determined from (7) with the use of just the observations from the subset $J(u^k, \bar{x}, \delta)$. If the number of observations, which are in $J(u^k, \bar{x}, \delta)$ is not enough for determining the parameters of the regression function (that depends on the number of the parameters of the model themselves), then the distance δ is increased. Note that it is not necessary to change the mathematical model at each iteration of the optimization method; it is enough to do it when u^{k+1} goes out of the sub-domain δ of the neighborhood u^k, in which the mathematical model was being built.

For the second variant of the suggested approach, the functions $y_i = y_i(u; x, p)$, namely the parameters p^k, are determined in the following way at the each iteration (6):

$$p^k = \arg\min_p \left\{ \sum_{j=1}^{N} \gamma^j \rho_x^j(\bar{x}) \rho_u^j(u^k) \left[y(\tilde{u}^j; \tilde{x}^j, p) - y^j \right]^2 \right\}, \tag{8}$$

where $\rho_x^j(\bar{x}), \rho_u^j(u^k)$ are weight functions determined by the distance of the observed values u^j, x^j from the current values $u^k, \bar{x}$. At that the longer the distance of the observed values from the current optimal value, the less the value of weight function.

The participation of the functions $\rho_x^j(\bar{x})$ and $\rho_u^j(u^k)$ in (6) is essential, as indeed their use distinguishes the suggested approach from the classical one.

The following functions can be chosen as the suggested weight functions:

$$\rho_x^j(\bar{x}) = \exp(-\gamma_1 \left\| x^j - \bar{x} \right\|_{E^r}, \quad x^j \in \tilde{X}, \tag{9'}$$

$$\rho_u^j(u^k) = \exp(-\gamma_2 \left\| \tilde{u}^j - u^k \right\|_{E^n} \quad \tilde{u}^j \in \tilde{U}, \quad j = 1, \dots, N \tag{9''}$$

where $\left\| \bullet \right\|_{E^r}$ is Euclidean norm in r-dimensional space, and γ_1, γ_2 are the given positive constants.

The procedure (8) is continued up to that moment when some condition of optimality or of halting of the iteration optimization method takes place with the given precision. Particularly, for instance, one of the given below equivalent conditions may serve as a termination criterion for the procedure (8):

$$\left\| u^{k+1} - u^k \right\|_{E^n} \le \varepsilon ,$$

$$\left| y_0(u^{k+1}; \bar{x}, p^{k+1}) - y_0(u^k; \bar{x}, p^k) \right| \le \varepsilon ,$$

$$\left\| S^k \right\|_{E^n} \le \varepsilon ,$$

$$\alpha_k = \arg\min_\alpha y_0(P_U(u^k + \alpha S^k), \bar{x}, p^k) \le \varepsilon .$$

Thus, in the suggested second variant of solving the decision-making problem, the whole set of statistical data about the technological process, but not the data from the sub-domains, which constitute the neighborhood of the optimum point, is used at each step of the iteration procedure of seeking the optimal value of the control and of building a mathematical model. Such variant of the suggested approach helps us in getting rid of solving the problems, linked with forming sub-domains for local approximation of the technological process, namely by selecting the quantity δ so that the number of observations would be enough for parametrical identification.

It is evident that the suggested approach, in contrast to the classical three-stage approach of decision-making, first of all requires more volume of the carried out calculations due to the necessity of parametrical identification at the each iteration of the optimization method. Secondly, the target functional and constraints vary at the each iteration of the optimization method, which may make difficult the process of carrying out optimization itself. The main positive factor of the suggested approach is that the made decision is optimal for the mathematical model, which was built taking into account the observations from the informational model the most approximate to the optimal solution, i.e. the mathematical model of the object itself can be considered as locally optimal with regard to the made decision.

Note that the approach suggested in the work uses the idea of adaptive statistical modeling, investigated in the work [5] and develops the results of the work [7]. It can be used successfully while modeling and optimizing the control by dynamical objects.

The numerical experiments carried out show the effectiveness of the suggested approach, at that the obtained optimal controls (regimes) may substantially differ from the controls obtained by classical sequential two-stage method of parametrical identification and optimization.

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