# APPLICATION OF SECOND ORDER METHODS FOR IDENTIFYING DOMAIN BOUNDARY

**Alexander Handzel[1], Anar Rahimov[2], and Mahir Malikov[3]**

[1]North Caucasus State Technical University, Stavropol, Russia, *hndalxvld@yahoo.com*
[2]Azerbaijan State Oil Academy, Baku, Azerbaijan, *anar_r@yahoo.com*
[3]Institute of Cybernetics of ANAS, Baku, Azerbaijan, *mr.malikov@mail.ru*

The physical problem investigated in this paper is shape optimization of heat conducting, elastic bodies. The novel aspect in the study lies in the application of the second order methods. We consider an elastic body occupying a domain $\Omega \subset E^2$ bounded by a boundary $\Gamma = \Gamma_1 \cup \Gamma_2$. The problem of domain boundary optimization is studied in the following formulation:

$$\frac{\partial u}{\partial t} = a^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + f(x, y, t), (x, y) \in \Omega, t \in (0, T), \quad (1)$$

$$u(x, y, 0) = \varphi_0(x, y), (x, y) \in \Omega, \quad (2)$$

$$u(x, y, t) = \varphi_1(x, y, t), (x, y) \in \Gamma_1, t \in (0, T), \quad (3)$$

$$u(x, y, t) = \varphi_2(x, y, t), (x, y) \in \Gamma_2, t \in (0, T), \quad (4)$$

$$mes\ \Omega = 1, \quad (5)$$

$$J(\Gamma_2, u, t) \to \min. \quad (6)$$

Here $f(x, y, t), \varphi_0(x, y)$, $\varphi_1(x, y, t)$ , $\varphi_2(x, y, t)$ are given functions, boundary $\Gamma_1$ is also given and boundary $\Gamma_2$ is to be found. We reduce initial problem (1)-(6) to a discrete problem. We assume that $\Gamma_2$ can be approximated by function:

$$G(x) = \sum_{i=1}^{n} \alpha_i \ \psi_i(x). \quad (7)$$

Coefficients $\alpha_i, i = 1,..., n$ are unknown optimized parameters. After applying (7) we obtain domain $\hat{\Omega}$. Then we introduce a net domain $\omega$ to approximate $\hat{\Omega}$. To solve boundary value problem (1)-(4) we use the schemes [1] and non-regular grid for near-boundary points of $\omega$, in particular. Thus we obtain finite-dimensional optimization problem. We calculate the gradient of objective functional of discretized problem, and when doing this we use the approach [2,3]. Then relying on ideas [4] we propose formulas to construct Hessian, that enables us to solve the formulated problem applying second order methods.

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## Literature

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